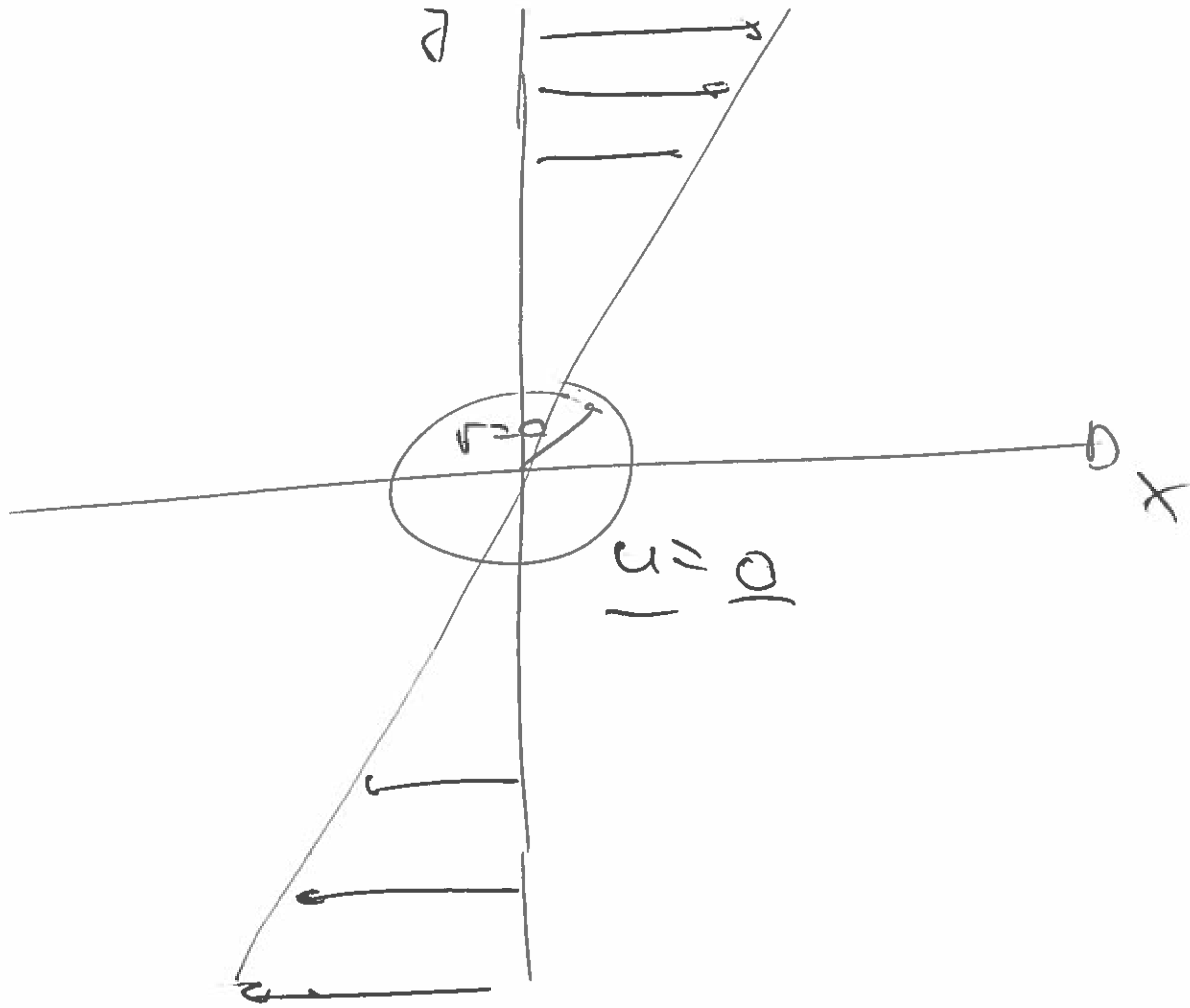


$$u_r \to \frac{1}{2}\dot{\gamma} r \sin(2\varphi) = \frac{1}{r}\frac{\partial\psi}{\partial\varphi} \quad \text{as } r \to \infty$$

$$u_\varphi \to -\frac{1}{2}\dot{\gamma} r \left(1-\cos(2\varphi)\right) = -\frac{\partial\psi}{\partial r}$$

$$\nabla^4\psi = 0$$

Given the φ-dependence of the far-field BCs: soln must have a part that is axisymmetric & a part that varies like $\cos(2\varphi)$.

Take relevant terms from formula sheet:

$$\psi(r,\varphi) = \cancel{A_0} + B_0 r^2 + C_0 \ln r + D_0 r^2 \ln r +$$
$$+\left(A_2 r^2 + B_2 r^{-2} + C_2 r^4 + D_2 r^0\right)\cos(2\varphi)$$

8 constants but only 4 BCs.
Set $A_0 = 0$ (arbitrary constant)

$$u_\varphi = -\frac{\partial\psi}{\partial r} = -2B_0 r - \frac{C_0}{r} - D_0\cancel{(2r\ln r + r)} +$$
$$+\left(-2A_2 r + 2B_2 r^{-3} - \cancel{4C_2 r^3}\right)\cos(2\varphi)$$

As $r \to \infty$:

$$u_\varphi \to -\frac{1}{2}\dot{\gamma} r\left(1 - \cos 2\varphi\right)$$

${}^{*}D_0$ must be zero because $r\ln r$ grows too quickly as $r \to \infty$.

r-dependent term:

$$-2B_0 = -\frac{1}{2}\dot{\gamma} \Rightarrow \boxed{B_0 = \frac{1}{4}\dot{\gamma}}$$

$$-2A_2 = \frac{1}{2}\dot{\gamma} \qquad \boxed{A_2 = -\frac{1}{4}\dot{\gamma}}$$

$\underline{r = a:}$ $u_\varphi(r=a) = 0$

$$\underbrace{\left(-2B_0 a - \frac{C_0}{a}\right)}_{=0} + \underbrace{\left(-2A_2 a + 2B_2\frac{1}{a^3}\right)}_{=0}\cos(2\varphi) = 0$$

$$C_0 = -2B_0 a^2 \qquad B_2 = a^4 A_2$$

$$\boxed{C_0 = -\frac{1}{2}a^2 S} \qquad \boxed{B_2 = -\frac{1}{4}a^4 S}$$

$$u_r = \frac{1}{r}\frac{\partial\psi}{\partial\varphi} = \left(-2A_2 r - 2B_2\frac{1}{r^3} - 2D_2\frac{1}{r}\right)\sin(2\varphi)$$

$\underline{BC:}$

$$u_r \to \frac{1}{2}Sr\sin(2\varphi) \quad \text{as} \quad r\to\infty$$

$$-2A_2 = \frac{1}{2}S \Rightarrow \boxed{A_2 = -\frac{1}{4}S}$$

(again)

$\underline{\text{Also:}}$ $u_r(r=a) = 0$

$$A_2 a + B_2\frac{1}{a^3} = -D_2\frac{1}{a}$$

$$\vdots$$

$$\boxed{D_2 = \frac{1}{2}a^2 S}$$

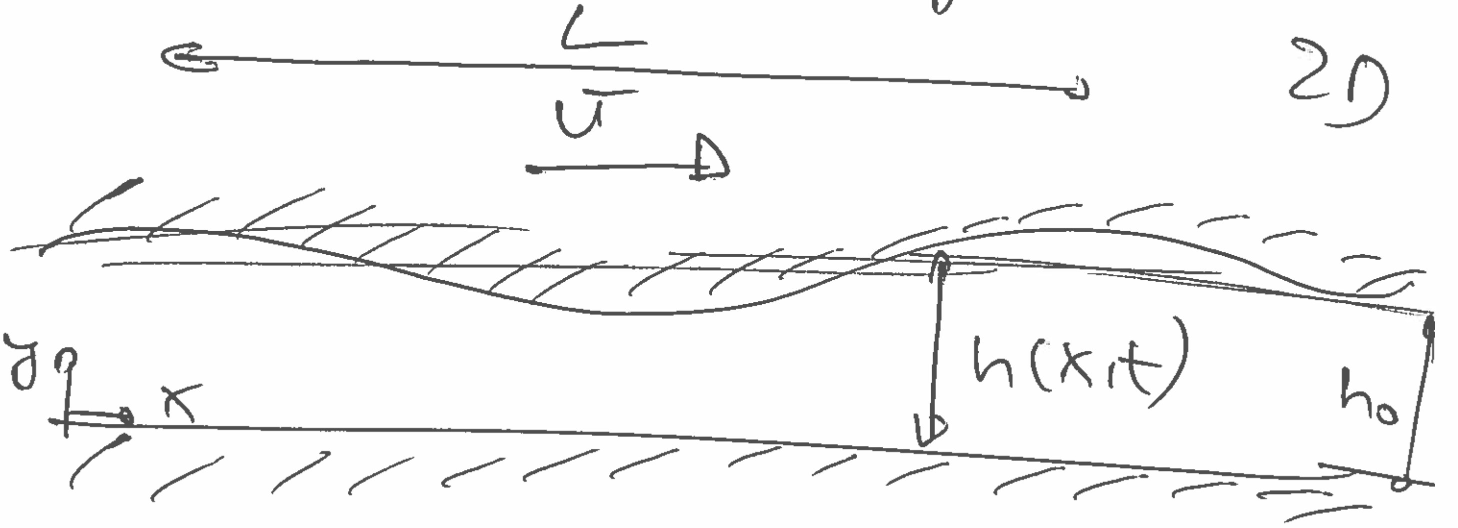
<u>So:</u>

$$\psi(r,\theta) = S\left\{\left(\frac{1}{4}r^2 - \frac{1}{2}a^2\ln(r)\right) + \left(-\frac{1}{4}r^2 - \frac{1}{4}a^4\frac{1}{r^2} + \frac{1}{2}a^2\right)\cos(2\theta)\right\}$$

(5

§N: Lubrication theory

Example for scaling



Gap is narrow & gently varying

$L \gg h_0$

↳ lengthscale over which the gap-width changes.

$\frac{h_0}{L} \ll 1$ use this to scale.

$x = L\tilde{x}$ $\qquad t = \frac{L}{U}\tilde{t}$

$y = h_0\tilde{y}$ $\qquad p = P\tilde{p}$

$u = U\tilde{u}$

$v = V\tilde{v}$

V & P are as yet unknown.

$h_0/L \ll 1$

<u>Continuity:</u>

$$\frac{U}{L}\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{V}{h_0}\frac{\partial \tilde{v}}{\partial \tilde{y}} = 0$$

Balance these terms: Choose:

$$V = \frac{h_0}{L}U \ll U.$$