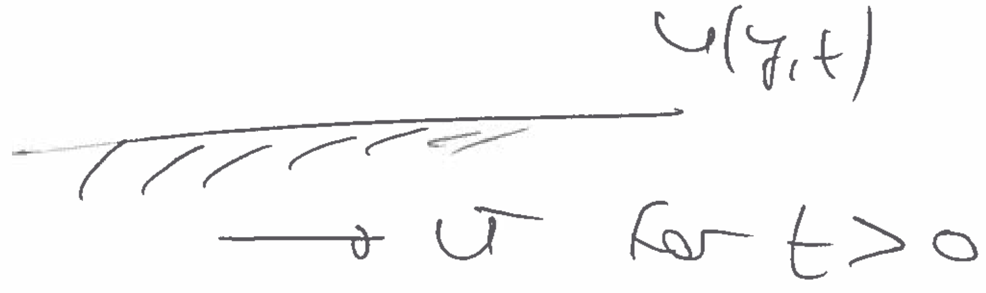


$$\boxed{\begin{aligned} &\cancel{\alpha}\frac{\partial \hat{u}}{\partial t} = \cancel{\alpha}\nu \frac{\partial^2 \hat{u}}{\partial y^2} \\ &\alpha \hat{u}\big|_{y=0} = \alpha U \\ &\alpha \hat{u} \to 0 \text{ as } y \to \infty \\ &\alpha \hat{u} = 0 \text{ at } t = 0 \end{aligned}}$$

Have EdU!

Assume soln for U is $\hat{u}$.
what is soln for αU?
Claim: $\alpha \hat{u}$

$$u(y,t;\nu,U) = U f(\underbrace{y,t;\nu}_{\text{dimless!}})$$

$$= U f(\eta)$$

$$\eta = \frac{y}{\sqrt{\nu t}}$$

$$\boxed{f'' + \tfrac{1}{2}\eta f' = 0}$$ (2)

$f = 1$ for $\eta = 0$

$f \to 0$ as $\eta \to \infty$ (2x)

Subst: $F = f'$

$$F' + \tfrac{1}{2}\eta F = 0$$

$$\frac{F'}{F} = -\tfrac{1}{2}\eta$$

$$\ln \frac{F}{F_0} = -\frac{1}{4}\eta^2$$

$$F = f' = F_0 \exp\left(-\tfrac{1}{4}\eta^2\right)$$

$$f(\eta) = A + F_0 \int^{\eta} \exp\left(-\tfrac{1}{4}\xi^2\right) d\xi$$

Lower limit? Choose ∞.

$$f(\eta) = A + B\int_{\eta}^{\infty} \exp\left(-\tfrac{1}{4}\xi^2\right) d\xi$$

$B = -f_0$

BC: $f \to 0$ as $\eta \to \infty$: $A = 0$

$$f(0) = 1 = B\underbrace{\int_{0}^{\infty} \exp\left(-\tfrac{1}{4}\xi^2\right) d\xi}_{\sqrt{\pi}}$$

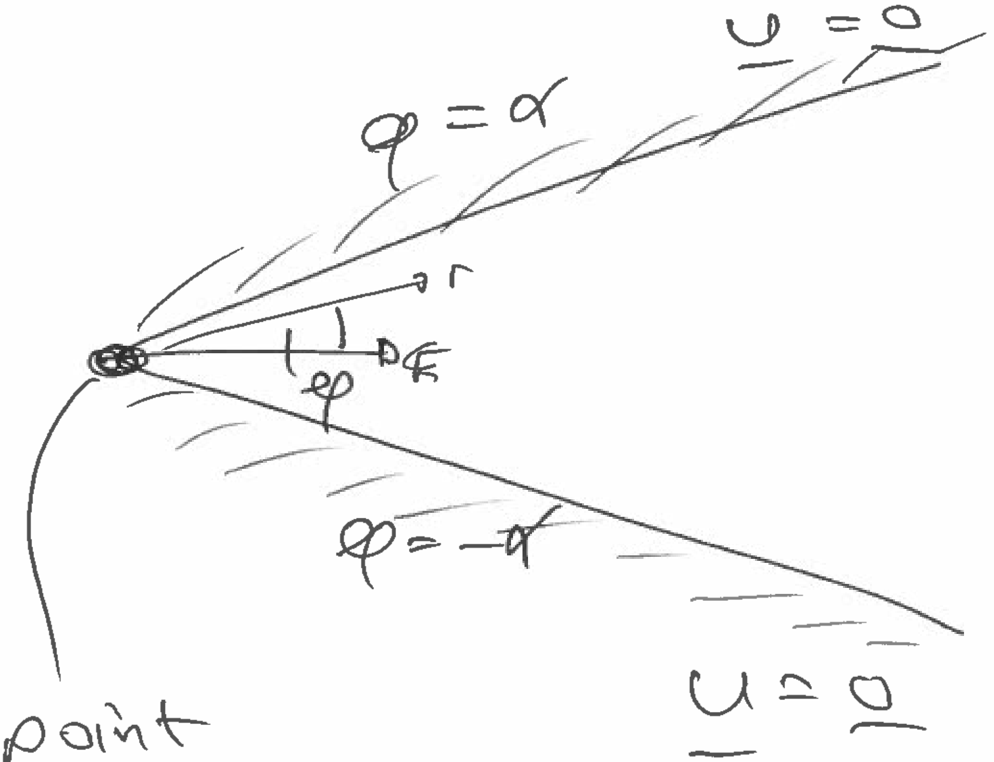
$$\boxed{u(y,t) = \frac{U}{\sqrt{\pi}}\int_{\eta}^{\infty} \exp\left(-\tfrac{1}{4}\xi^2\right) d\xi \quad \text{where } \eta = \frac{y}{\sqrt{\nu t}}}$$

Also:

$$u = U\,\mathrm{erfc}\left(\frac{\eta}{2}\right)$$

Example: Jeffrey-Hamel flow

2D



point sink: Q ← volume flux per unit depth in z-direction
(syringe).

$$\underline{u} = \underline{u}(r, \varphi; \rho, \mu, Q, \alpha)$$

Dimensions:

ρ	kg/m^3
ν	m^2/sec
Q	m^2/sec
α	1
φ	1
r	m
$\underline{u}$	m/sec

To get dimensions of $\underline{u}$ correct

$$\underline{u} = \frac{\nu}{r}\,\underline{f}(r, \varphi; \rho, \nu, Q, \alpha)$$

<u>Note:</u> Q/r would have worked too.

$\underline{f}$ must be dim.less fct. of its arguments

$\Rightarrow$ $\underline{f}$ cannot depend on ρ!

$\Rightarrow$ Q & ν must combine to Q/ν

$\Rightarrow$ r cannot appear

$$\underline{u} = \frac{\nu}{r}\, \underbrace{\underline{f}\left(\varphi; \alpha, \frac{Q}{\nu}\right)}$$

~~Continuity:~~ $\underline{f}(\varphi) = f(\varphi)\,\underline{e_r} + \cancel{g(\varphi)\,\underline{e_\varphi}}$

$$\frac{1}{r}\frac{\partial}{\partial r}(r u) + \frac{1}{r}\frac{\partial v}{\partial \varphi} = 0$$

$$\cancel{\frac{1}{r}\frac{\partial}{\partial r}\left(\nu f(\varphi)\right)} + \frac{1}{r}\frac{\partial}{\partial \varphi}\left(\frac{\nu}{r} g(\varphi)\right) = 0$$

$$\frac{\partial g}{\partial \varphi} = 0$$

recall g is azimuthal component of veloc.

$v = \text{const}$

Apply BC: $\underline{u} = \underline{0}$ at $\varphi = \pm\alpha$

$\Rightarrow v \equiv 0$

$\Rightarrow$ pure radial flow.