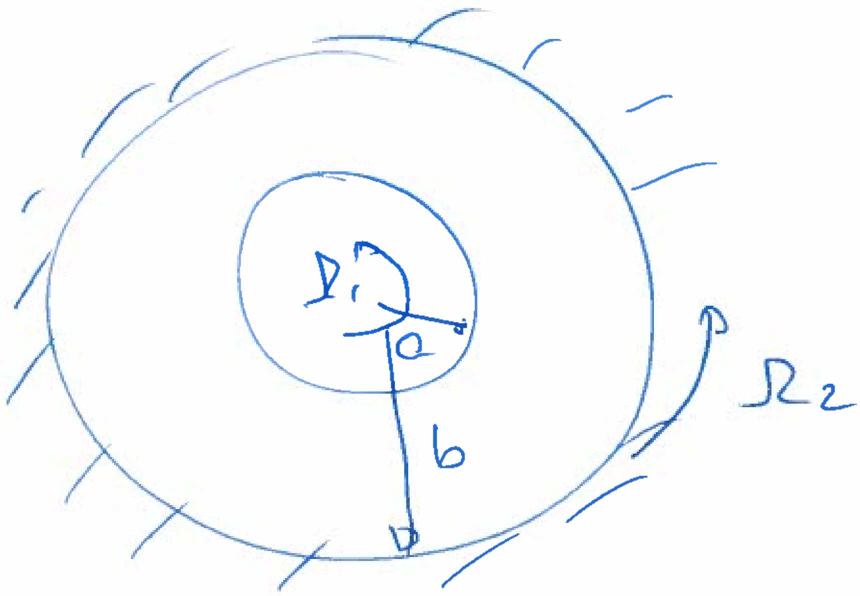


$\underline{u} = \omega(r)\,\underline{e}_\varphi$ "$\nabla p = \underline{0}$"

$$r^2 \frac{\partial^2 u}{\partial r^2} + r \frac{\partial u}{\partial r} - u = 0$$

Euler ODE

Ansatz:

$$u(r) \sim r^\lambda$$

into ODE:

$$r^\lambda \underbrace{\left(\lambda(\lambda-1) + \lambda - 1\right)}_{=0} = 0$$

$$\lambda^2 \cancel{-\lambda} + \cancel{\lambda} - 1 = 0$$

$$\lambda = \pm 1$$

Gen. soln:

$$v(r) = Ar + \frac{B}{r}$$

A, B from BC:

$$v(r=a) = a\Omega_1$$
$$v(r=b) = b\Omega_2$$

(No slip)

$\vdots$

$$v(r) = \frac{1}{b^2-a^2}\left\{(b^2\Omega_2 - a^2\Omega_1)r - \frac{a^2b^2(\Omega_2-\Omega_1)}{r}\right\}$$

Check: $\Omega_1 = \Omega_2 = \Omega$

$$v(r) = \Omega r \quad \checkmark$$

(rigid body rotation)

But still have to check the radial momentum eqn.

$$-\frac{v^2}{r} = 0 \quad ?$$

Inconsistent! One of our assumptions must have been wrong:
pressure must vary with r (centrifugal effects)!

Luckily rest of analysis stays unchanged.

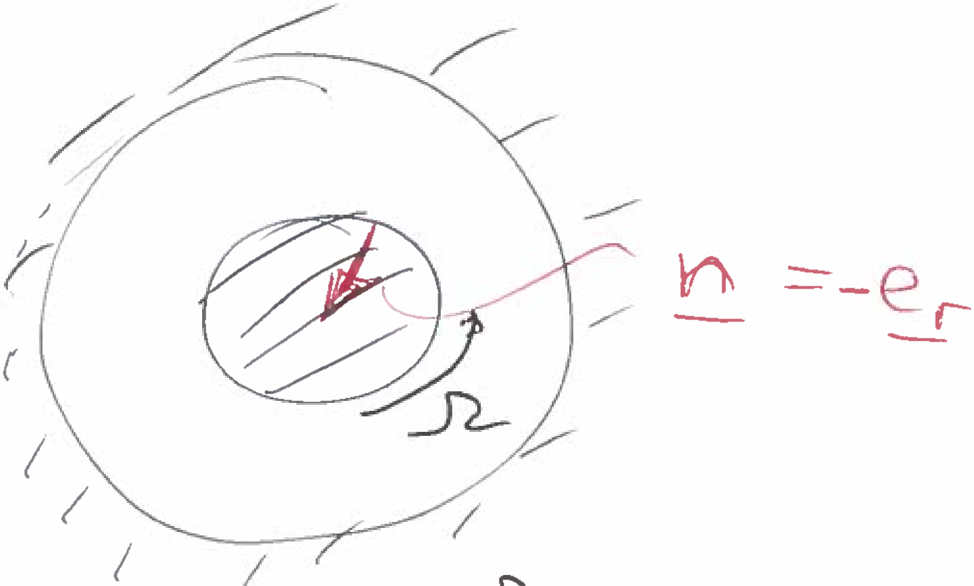
r-mom. eqn:

$$\frac{dp}{dr} = \rho \frac{u^2}{r}$$ unchanged!

integrate to obtain the pressure.

Traction on inner cylinder

For simplicity $\Omega_1 = \Omega$ $\Omega_2 = 0$



$$v(r) = \frac{a^2\Omega}{b^2-a^2}\left(\frac{b^2}{r} - r\right)$$

Traction inner cylinder exerts onto fluid?

$$t_i = \tau_{ij} n_j$$

$$\tau_{ij} = -p\,\delta_{ij} + 2\mu e_{ij}$$

$i, j = \{r, \varphi, z\}$

$\hookrightarrow \equiv \varepsilon_{ij}$

$$\underline{n} = n_r \underline{e_r} + n_\varphi \underline{e_\varphi} + n_z \underline{e_z} = -\underline{e_r}$$

$$n_r = -1, \quad n_\varphi = n_z = 0$$

$$u = w = 0, \quad v = v(r)$$

e_{ij} : All zero apart from

$$\epsilon_{rr} = \cancel{\frac{\partial u}{\partial r}} \qquad \epsilon_{\varphi\varphi} = \cancel{\frac{1}{r}\frac{\partial v}{\partial \varphi}} + \cancel{\frac{u}{r}}$$

$$\epsilon_{zz} = \cancel{\frac{\partial w}{\partial z}} \qquad \epsilon_{r\varphi} = \frac{1}{2}\left[r\frac{\partial}{\partial r}\left(\frac{v}{r}\right) + \cancel{\frac{1}{r}\frac{\partial u}{\partial \varphi}}\right]$$

$$\epsilon_{\varphi z} = \frac{1}{2}\left[\cancel{\frac{1}{r}\frac{\partial w}{\partial \varphi}} + \cancel{\frac{\partial v}{\partial z}}\right] \qquad \epsilon_{rz} = \frac{1}{2}\left[\cancel{\frac{\partial u}{\partial z}} + \cancel{\frac{\partial w}{\partial r}}\right]$$

$$e_{r\varphi} = \frac{1}{2} r \frac{\partial}{\partial r}\left(\frac{v}{r}\right)$$

$$\vdots$$

$$e_{r\varphi} = -\frac{a^2 b^2 \Omega}{b^2-a^2} \frac{1}{r^2}$$

$$e_{r\varphi}\Big|_{r=a} = -\frac{b^2 \Omega}{b^2-a^2}$$

$$t_i = -p n_i + 2\mu e_{ij} n_j$$

$i =$ "r"

$$t_r = -p \overset{-1}{n_r} + 2\mu\left(\cancel{e_{rr} n_r} + e_{r\varphi}\cancel{n_\varphi} + e_{rz}\cancel{n_z}\right)$$

$$\boxed{t_r = p\,(r=a)}$$

$i =$ "φ"

$$t_\varphi = -p \cancel{n_\varphi} + 2\mu\left(e_{\varphi r} \overset{-1}{n_r} + e_{\varphi\varphi}\cancel{n_\varphi} + e_{\varphi z}\cancel{n_z}\right)$$

$$\boxed{t_\varphi = 2\mu \frac{b^2\Omega}{b^2-a^2}} > 0$$

$$\boxed{t_z = 0}$$

$$\boxed{t_r = p(r=a)}$$