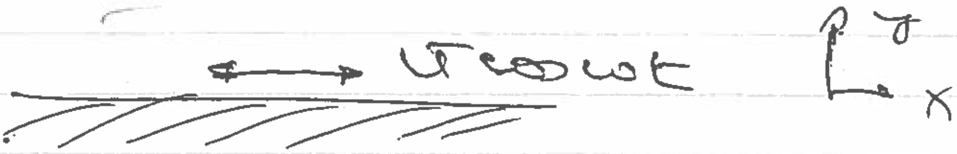
VISUAL USER FAILURES: THIS WAS ON BLACKBOARD

Example: The vibrating plate:



No body force.

Flow driven by wall motion (~ Couette flow)

expect parallel flow in x-direct.

$\frac{\partial p}{\partial x} = G = 0$; but time-dep.

$$\boxed{\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2}} \qquad u = u(y,t)$$

BC.

$u = U\cos\omega t$ @ $y = 0$

$u \to 0$ as $y \to \infty$

Look for periodic steady state soln.

$u(y,t) = f(y)\cos(\omega t + \ldots)$

easier (as always) using complex vars.

$$u(y,t) = f(y)\, e^{i\omega t}$$

into $\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2}$

$$i\omega f = \nu f''$$

$$f'' - \frac{i\omega}{\nu} f = 0$$

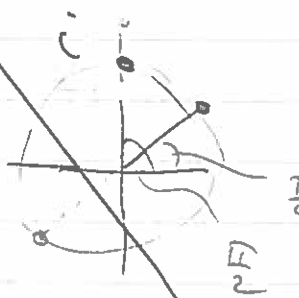
linear ODE with const. coeffs

$$f \sim e^{\lambda y}$$

$$\lambda^2 - \frac{i\omega}{\nu} = 0$$

$$\lambda = \pm\sqrt{\frac{i\omega}{\nu}}$$

$$\sqrt{i} = \frac{1}{2}\sqrt{2}\,(1+i)$$



$$\lambda = \pm(1+i)\sqrt{\frac{\omega}{2\nu}}$$

(neg. root is already in ±)

(1

$$u(y,t) = f(y)\, e^{i\omega t}$$

into PDE:

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2}$$

$$i\omega f = \nu f''$$

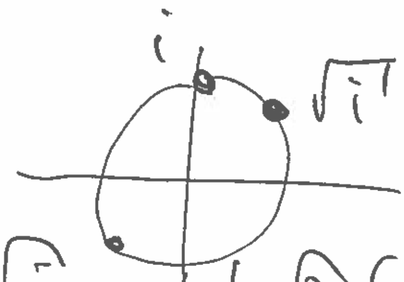
$$f'' - \frac{i\omega}{\nu} f = 0$$

ODE for $f(y)$

$$f(y) \sim e^{\lambda y}$$

$$\lambda^2 - \frac{i\omega}{\nu} = 0 \qquad \text{char. poly.}$$

$$\lambda = \pm\sqrt{\frac{i\omega}{\nu}} = \pm\sqrt{i}\sqrt{\frac{\omega}{\nu}}$$



$\sqrt{i} = \frac{1}{2}\sqrt{2}(1+i)$

$\sqrt{i} = -\frac{1}{2}\sqrt{2}(1+i)$

$$\lambda = \pm (1+i)\sqrt{\frac{\omega}{2\nu}}$$

$$f(y) = A e^{(1+i)\sqrt{\frac{\omega}{2\nu}}\, y} + B e^{-(1+i)\sqrt{\frac{\omega}{2\nu}}\, y}$$

BC:

$$f(0) = U = A + B$$

$$f \to 0 \text{ as } y \to \infty : \quad A = 0$$

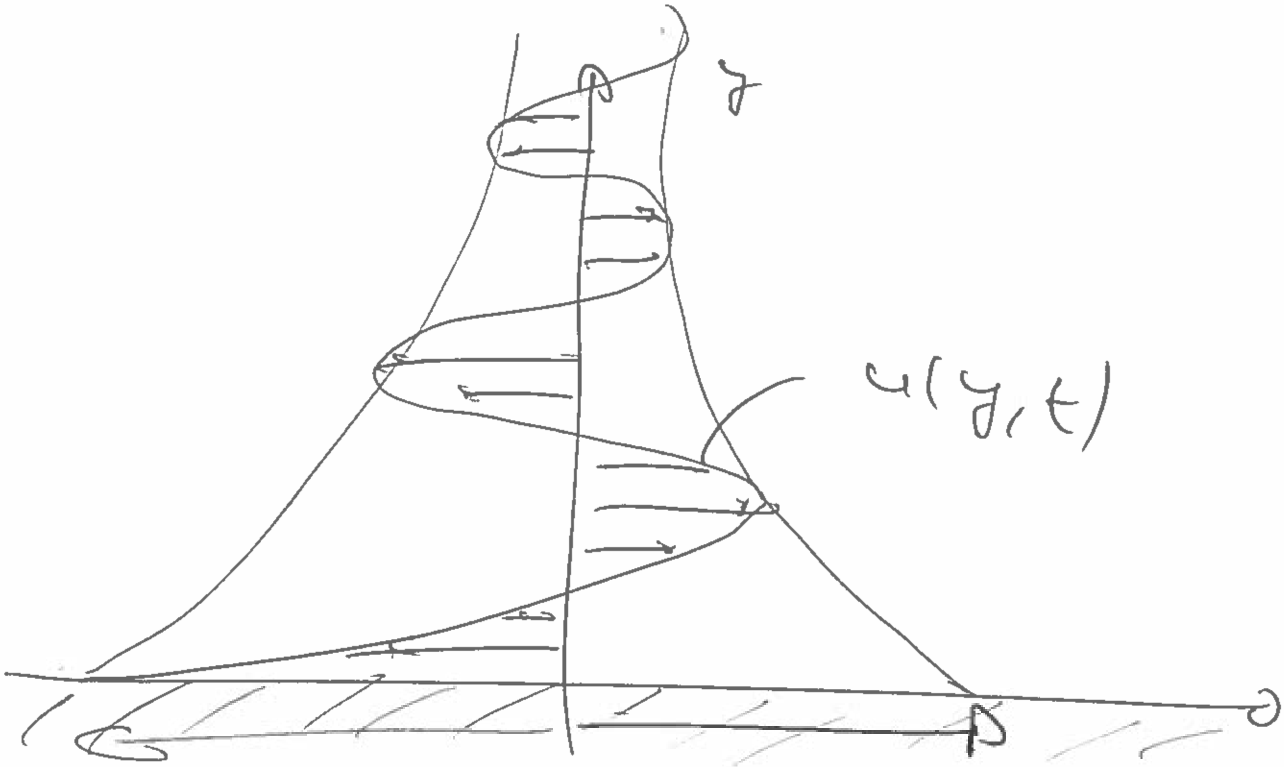
$$B = U$$

$$u(y,t) = \mathrm{Re}\left(U e^{-(1+i)\sqrt{\frac{\omega}{2\nu}}\, y}\; e^{i\omega t} \right)$$

convert to real:

$$u(y,t) = U e^{-\sqrt{\frac{\omega}{2\nu}}\, y} \cos\left(\omega t - \sqrt{\frac{\omega}{2\nu}}\, y\right)$$

BC ✓



See animation on source webpage.

§ 6 (?) Curvilinear coordinates

So far: Derived eqns. in Cartesian coords. (x_1, x_2, x_3)

$$\underline{u} = u_1 \underline{e_1} + u_2 \underline{e_2} + u_3 \underline{e_3}$$

Can choose different coordinate systems & transform differential operators. Example: 2D

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}$$

or: $x = r \cos\varphi \quad y = r \sin\varphi$

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \varphi^2}$$

for scalar field!

N. St. eqns contain derivatives of vectors!

$$\underline{u} = u_r \underline{e_r} + u_\varphi \underline{e_\varphi} + u_z \underline{e_z}$$

when evaluating derivs. of $\underline{u}$, we also have differentiate the basis vectors!

$\Rightarrow$ A mess!

But for orthogonal coord systems some eqns stay the same:

$$t_i = \tau_{ij} n_j \qquad i,j = [r, \varphi, z]$$

$$\tau_{ij} = -p\delta_{ij} + 2\mu \varepsilon_{ij}$$

ε_{ij} from formula sheet!