MATH35001: EXAMPLE SHEET[1] II

1.) A 2D flow field is given by $u_1 = ax_2$ and $u_2 = ax_1$, where $a > 0$ is a constant.

a) Sketch the flow field (begin by considering the velocity vectors on the coordinate axes and on the main diagonals).

b) Determine the trajectories $x_i^p(t)$ of particles which are at position $x_i = X_i$ at time $t = 0$ by integrating the equations of motion $\partial x_i^p(t)/\partial t = u_i(x_j^p(t), t)$.

c) Determine the acceleration of material particles directly from their trajectories (i.e. by evaluating $a_i = \partial^2 x_i^p(t)/\partial t^2$).

d) Compare this to the result obtained by using the material derivative $a_i = Du_i/Dt$.

e) What is the divergence of the flow field?

2.) A 2D flow field is given by $u_1 = Ux_2$ and $u_2 = 0$, where $U > 0$ is a known constant.

a) Sketch the flow field in the region $x_2 \in [0, 1]$.

b) Determine the rate of strain and the rate of rotation tensors ϵ_{ij} and ω_{ij}.

c) Sketch the deformation of a small rectangular fluid element whose edges of lengths δx_1 and δx_2 are parallel to the x_1 and x_2 axes, respectively. [**Hint**: Sketch the corners of the fluid element at time t and at time $t + \delta t$.]

Relate this to the interpretation of the rate of strain and rate of rotation tensors. [**Hint**: To interpret the entries in the rate of rotation tensor, consider the average of the rate at which the lines that were initially parallel to the x_1 and x_2 axes rotate about the x_3-axis.]

Coursework

Please exchange your solution to question 2 with your "marking buddy" and assess each other's work, using the master solution made available on the course webpage (probably in week 3).

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