$$\frac{\partial \tau_{ij}}{\partial x_j} + \rho F_i = \rho \frac{D u_i}{D t}$$

Cauchy's eqn.

(= Newton's law for continuum)

4. Symmetry of stress tensor

$$\tau_{ij} = \tau_{ji}$$

5. Constitutive eqns

Need: Link between τ_{ij} & the flow u_i.

Requires experiments.

Observations:

Fluids:

- can generate hydrostatic pressures.
- have a resistance to shear (knife in honey). Viscosity.

- do not generate internal stresses when subjected to rigid body motions.

$\Rightarrow$ τ_{ij} should have a hydrostatic press. field & depend on ε_{ij}.

A wide range of incompressible fluids (Newtonian fluids) behave according to:

$$\tau_{ij} = -p\,\delta_{ij} + 2\mu\,\varepsilon_{ij}$$

constant: "dynamic viscosity" → property of fluid.

$$\tau_{ij} = -p\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Insert into Cauchy:

$$\rho\frac{Du_i}{Dt} = \rho f_i + \frac{\partial \tau_{ij}}{\partial x_j}$$

$$= \rho f_i + \frac{\partial}{\partial x_j}\left(-p\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right)$$

$$= \rho f_i - \frac{\partial p}{\partial x_i} + \mu\frac{\partial^2 u_i}{\partial x_j \partial x_j} + \cancel{\mu\frac{\partial}{\partial x_i}\left(\frac{\partial u_j}{\partial x_j}\right)}$$

$\nabla \cdot \underline{u} = 0$

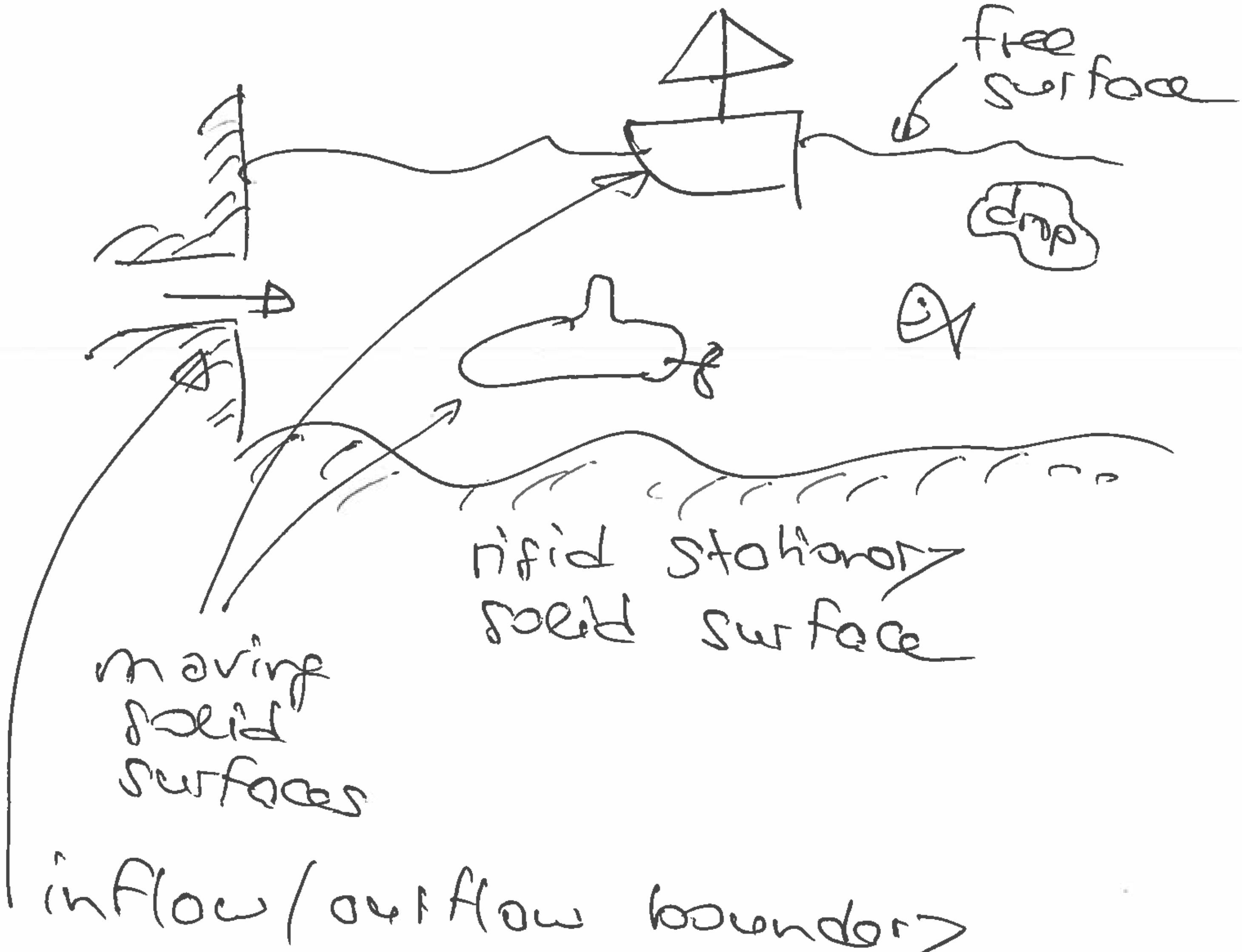
for incompr. fluids.

$$\rho\left(\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}\right)=\rho f_i-\frac{\partial p}{\partial x_i}+\mu\frac{\partial^2 u_i}{\partial x_j^2}$$

$$\frac{\partial u_j}{\partial x_j}=0$$

System of four nonlinear 2^{nd} order in space, 1^{st} order in time parabolic PDE for the three veloc. comp. & the pressure.

§4. Boundary & initial cond.



Initial conditions

Have to specify $u_i(x_j, t=0)$

Note: No I.C. for pressure!

Boundary conditions:

(i) inflow/outflow BC

$\boxed{u_i = U_i}$ (prescribed) on the inflow boundary

(ii) on ~~#~~ solid surfaces:

"No slip & no penetration"

fluid veloc. = solid veloc.

given

$$\boxed{u_i = U_i}$$

If solid is at rest:

$$\boxed{u_i = 0}$$