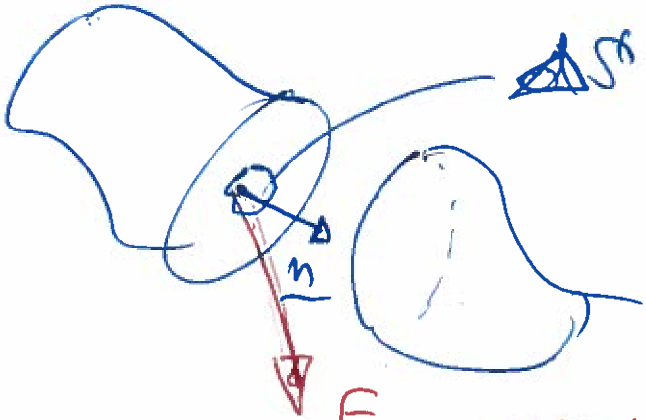


$\underline{F}$ = resultant force on ΔS

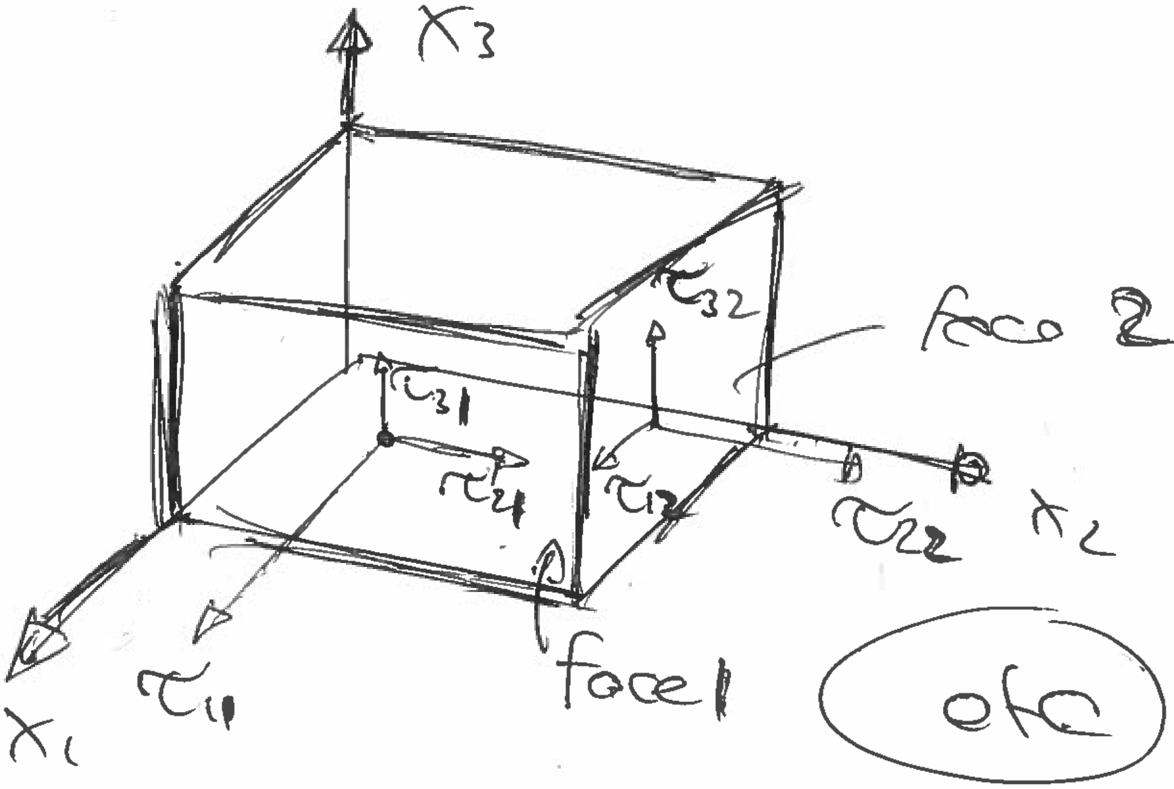
$$\boxed{\underline{t} = \lim_{\Delta S \to 0} \frac{\underline{F}}{\Delta S}}$$

Stress depends on orientation, $\underline{n}$

$$t_i = \underbrace{\tau_{ij}}_{\text{stress tensor}} n_j$$

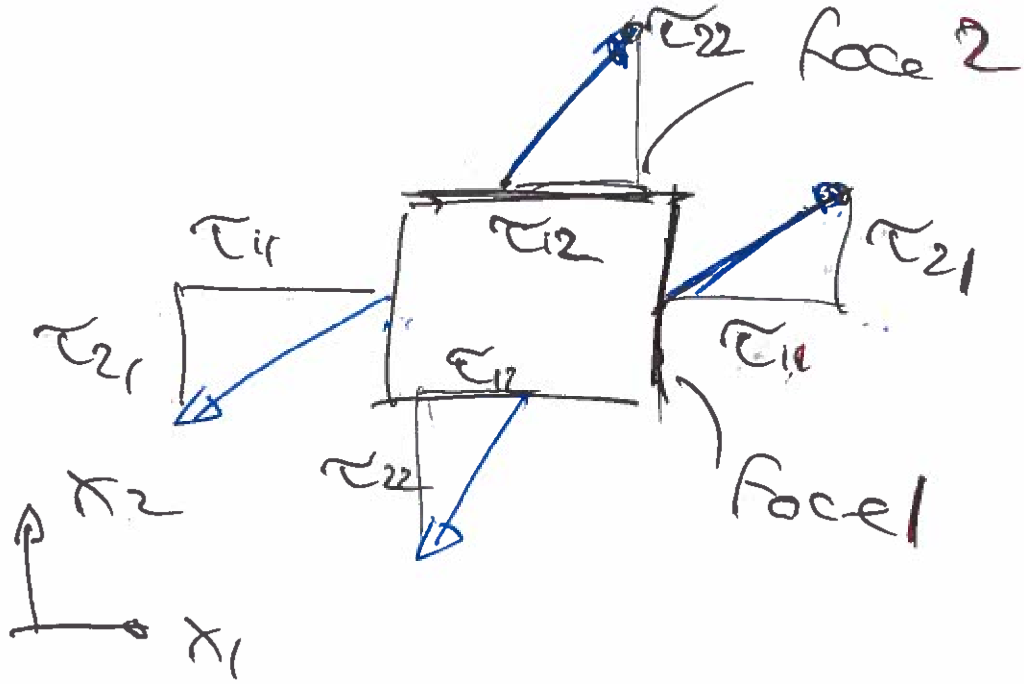
= i-th component of stress on face where x_j = const & outer unit normal points in pos. x_j-direction

Sketch:



Diagonal entries in τ_{ij} are normal components (pressure, tension)
off diagonal entries = tangential components = shear stresses
What ~~about~~ about the other faces? The only difference is the sign of the outer unit normal.

2D sketch:



Can now express the stress on a face in the fluid in terms of τ_{ij}.

Later we will determine τ_{ij} as a fct. of the fluid's kinematics.

Particular stress states

(i) Hydrostatic pressure

$$\tau_{ij} = -p\,\delta_{ij}$$

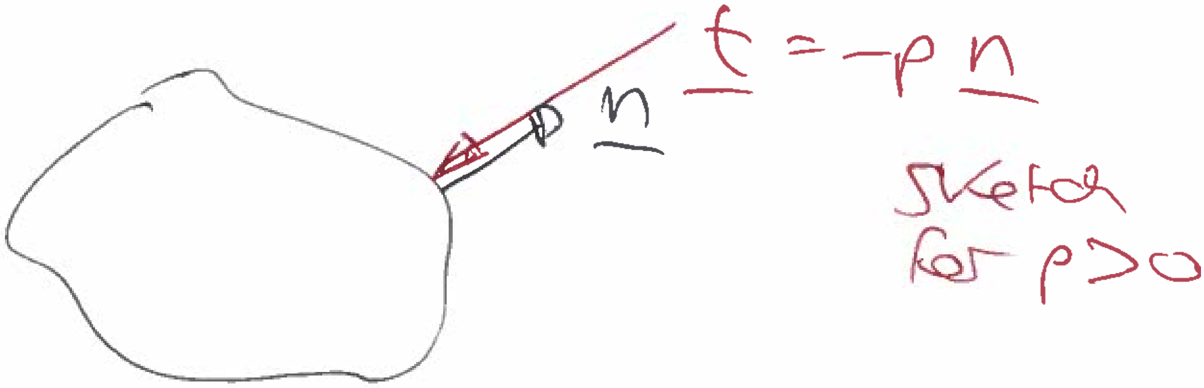
implies that the traction is always normal to the face & uniform in all directions.

Proof:

$$t_i = \tau_{ij} n_j$$

$$t_i = -p\,\delta_{ij} n_j = -p n_i$$

$$\underline{t} = -p\,\underline{n}$$

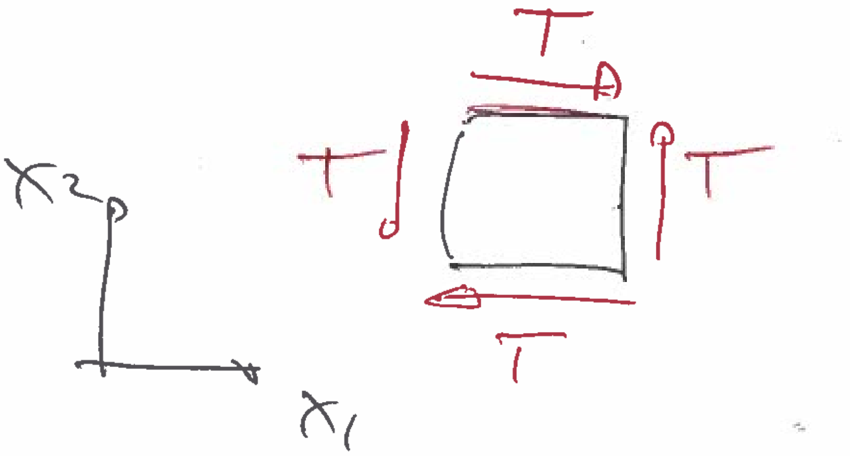


Also note: negative normal (σ
stress = compression.

(ii) pure shear stress

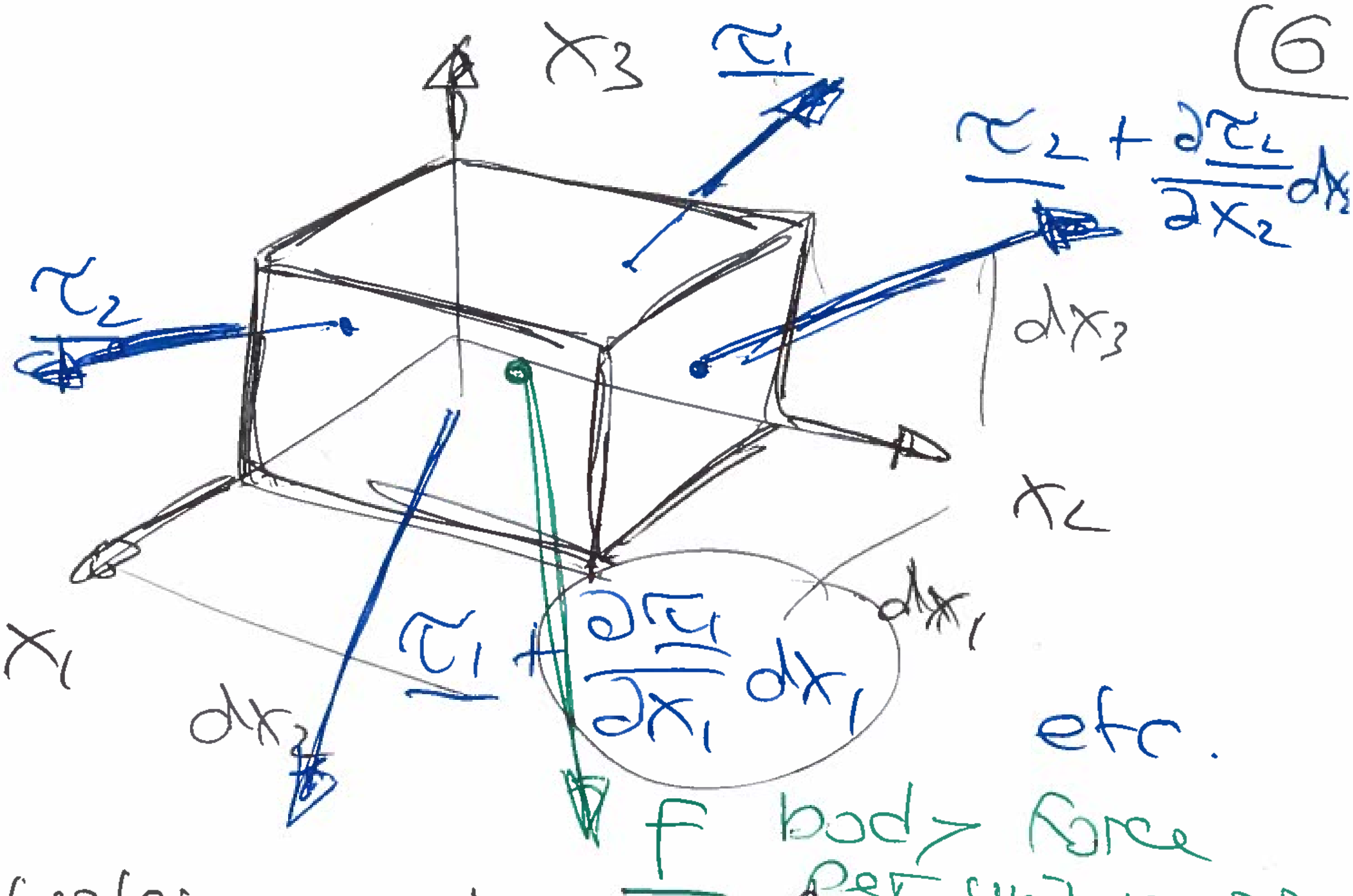
e.g. $\tau_{12} = \tau_{21} = T$

$\tau_{11} = \tau_{22} = 0$



3. Equilibrium of forces

" Σ forces = mass x accel."
for a small blob of fluid.



Note: contributions from opposite forces cancel, leaving just the increments.

$$\frac{\partial \underline{\tau}_1}{\partial x_1} \cancel{dx_1 \underbrace{dx_2 dx_3}_{\text{area}}} +$$

$$\frac{\partial \underline{\tau}_2}{\partial x_2} \cancel{dx_2 \underbrace{dx_1 dx_3}_{\text{area}}} +$$

$$\frac{\partial \underline{\tau}_3}{\partial x_3} \cancel{dx_3 \underbrace{dx_1 dx_2}_{\text{area}}} +$$

$$\underline{F} \rho \cancel{dx_1 dx_2 dx_3} = \rho \frac{D\underline{u}}{Dt} \cancel{dx_1 dx_2 dx_3}$$

Volume

summ. conv. & def. of τ_{ij}:

$$\boxed{\frac{\partial \tau_{ij}}{\partial x_j} + \rho f_i = \rho \frac{Du_i}{Dt}}$$

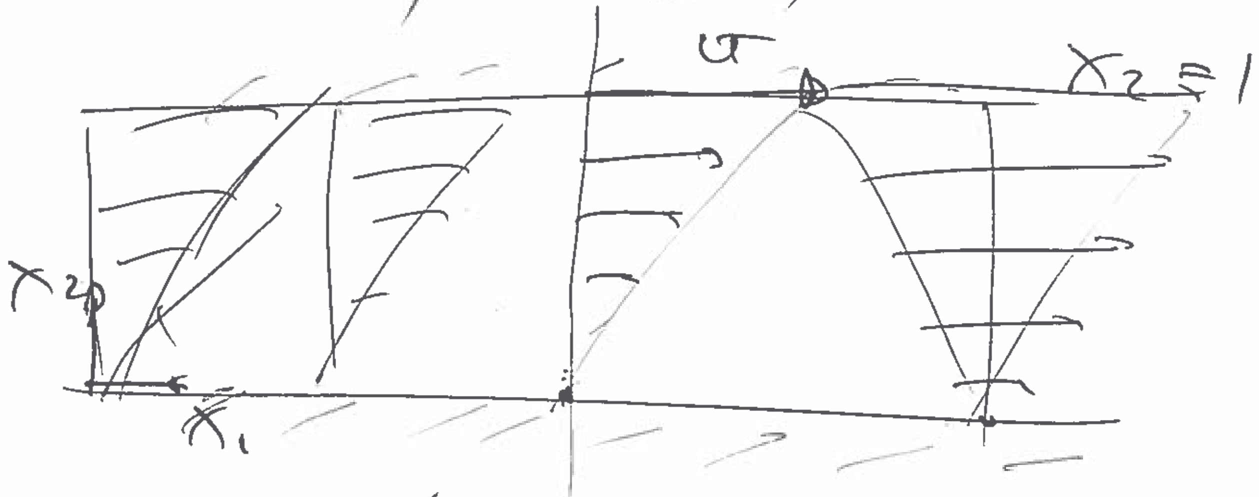
Cauchy's eqn!

X-CUTS)

(1

$$\underline{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} \sigma x_2 \\ 0 \end{pmatrix}$$

$\sigma > 0$



$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \qquad \omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right)$$

$$[\varepsilon_{ij}] = \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{21} & \varepsilon_{22} \end{pmatrix}$$

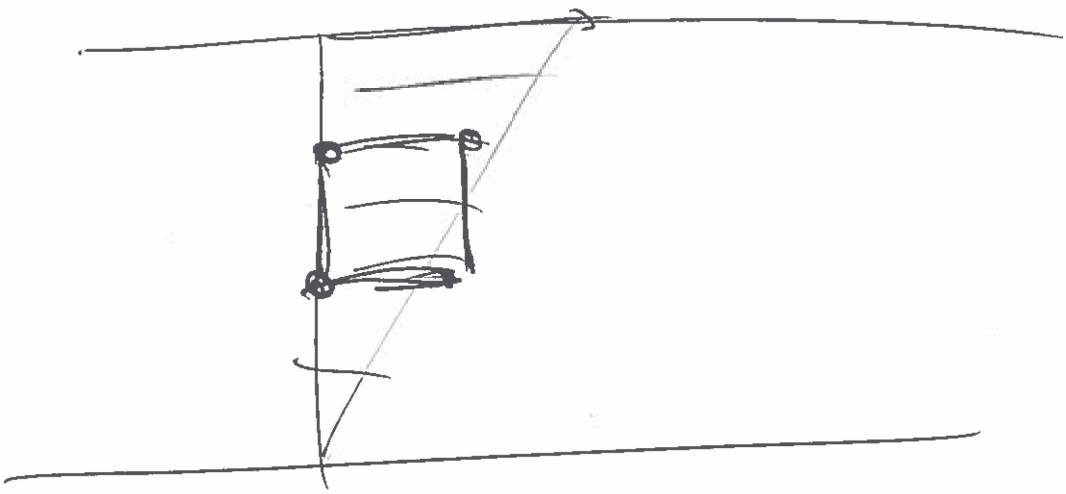
$$\begin{pmatrix} \frac{\partial u_1}{\partial x_1} & \frac{\partial u_1}{\partial x_2} \\ \frac{\partial u_2}{\partial x_1} & \frac{\partial u_2}{\partial x_2} \end{pmatrix} = \begin{pmatrix} 0 & \sigma \\ 0 & 0 \end{pmatrix}$$

$$\begin{array}{cc} & \quad j \\ i & \end{array}$$

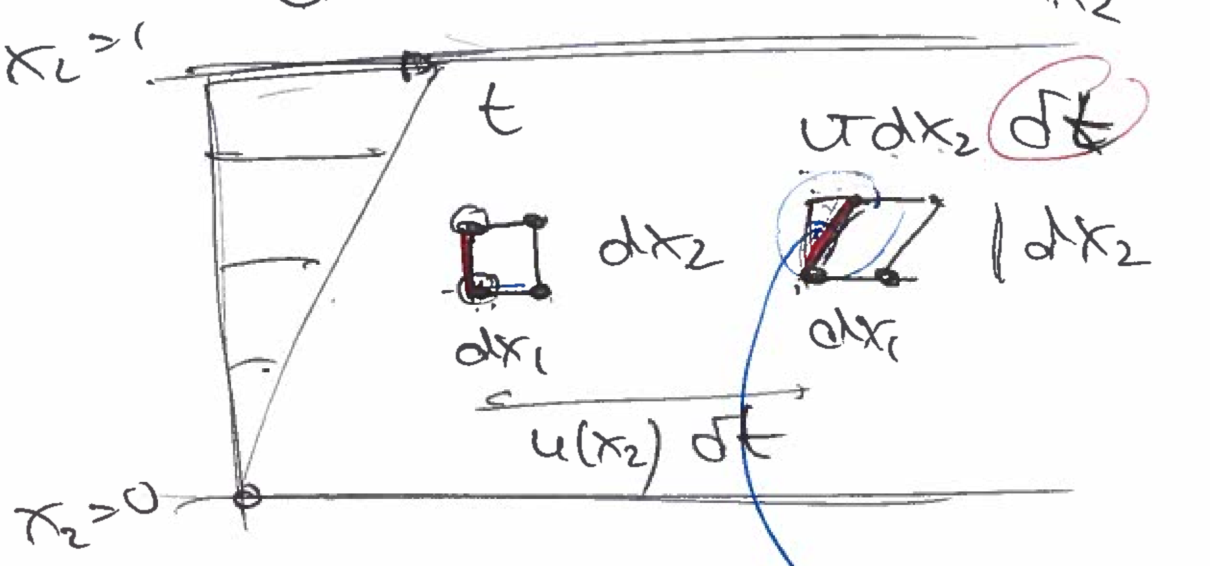
$$[\varepsilon_{ij}] = \begin{pmatrix} 0 & \frac{1}{2}u' \\ \frac{1}{2}u' & 0 \end{pmatrix}$$

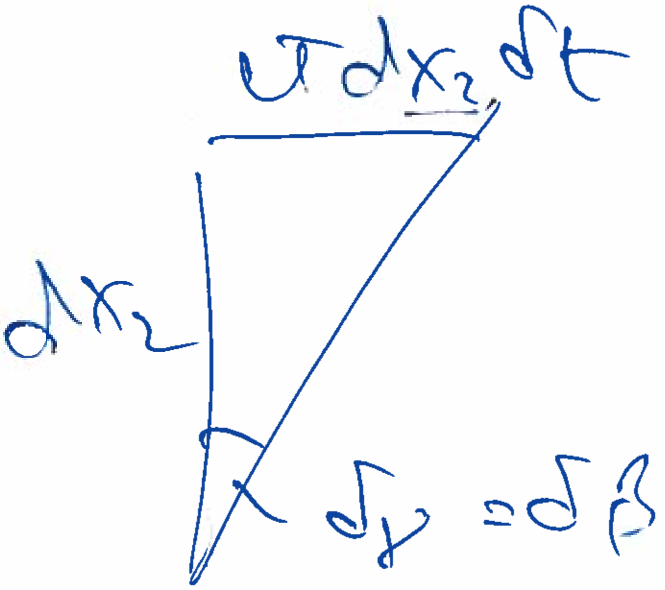
$$[\omega_{ij}] = \begin{pmatrix} 0 & \frac{1}{2}u' \\ -\frac{1}{2}u' & 0 \end{pmatrix}$$

Ω_3



$u_1 = U x_2$





$$\underbrace{\tan \delta\beta}_{\delta\beta} = \frac{U \cancel{dx_2}\, \delta t}{\cancel{dx_2}}$$

$$\frac{\delta\beta}{\delta t} = U$$

Av. rate of rotation

av. rate of rotation $= \frac{1}{2}\frac{d\beta}{dt}$

$= \omega$