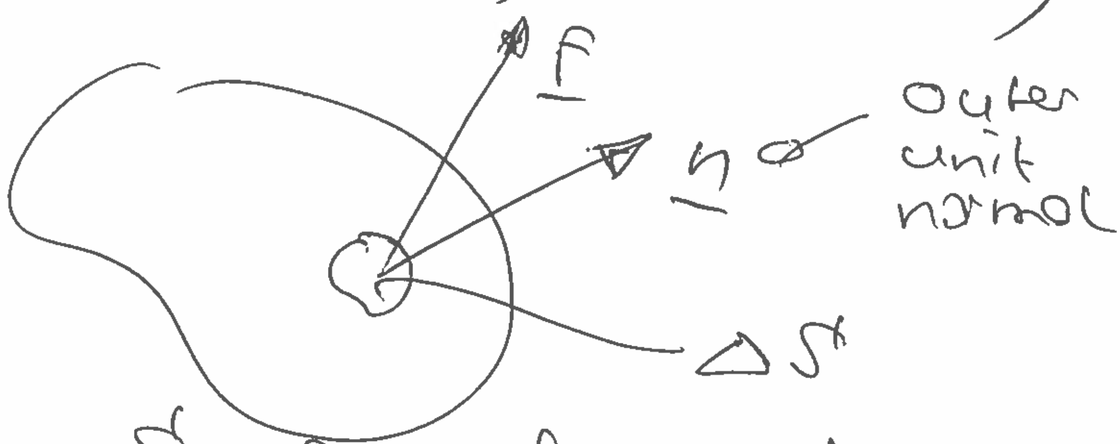
§3 Stress, Cauchy's eqn & the Navier-Stokes eqns

So far: Kinematics

Now: forces & equilibrium, equations of motion

(1) The concept of stress/traction

Consider a "blob" of fluid loaded by some distributed force (pressure, shear stress)



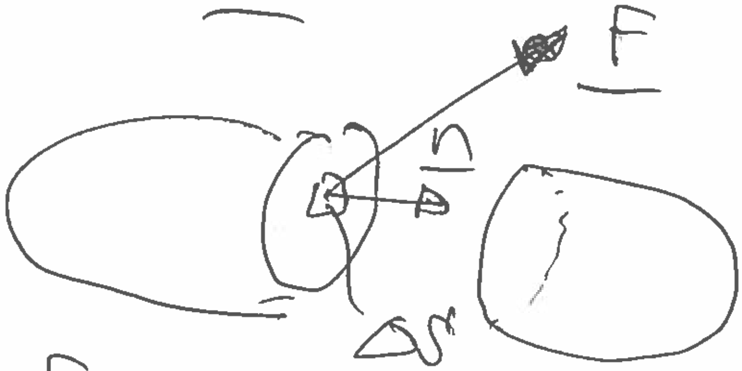
Patch ΔS of surface with outer unit normal $\underline{n}$ is subject to a resultant force $\underline{F}$.

Def: Traction

$$\boxed{\underline{t} = \lim_{\Delta S \to 0} \frac{\underline{F}}{\Delta S}}$$ vector!

$\underline{t}$ is force per unit area acting <u>onto</u> fluid.

Similarly: Cut the blob along some plane with normal $\underline{n}$



Here $\underline{F}$ = resultant force exerted onto ΔS by the "other half" of fluid.

Def: Stress

$$\boxed{\underline{t} = \lim_{\Delta S \to 0} \frac{F}{\Delta S}}$$

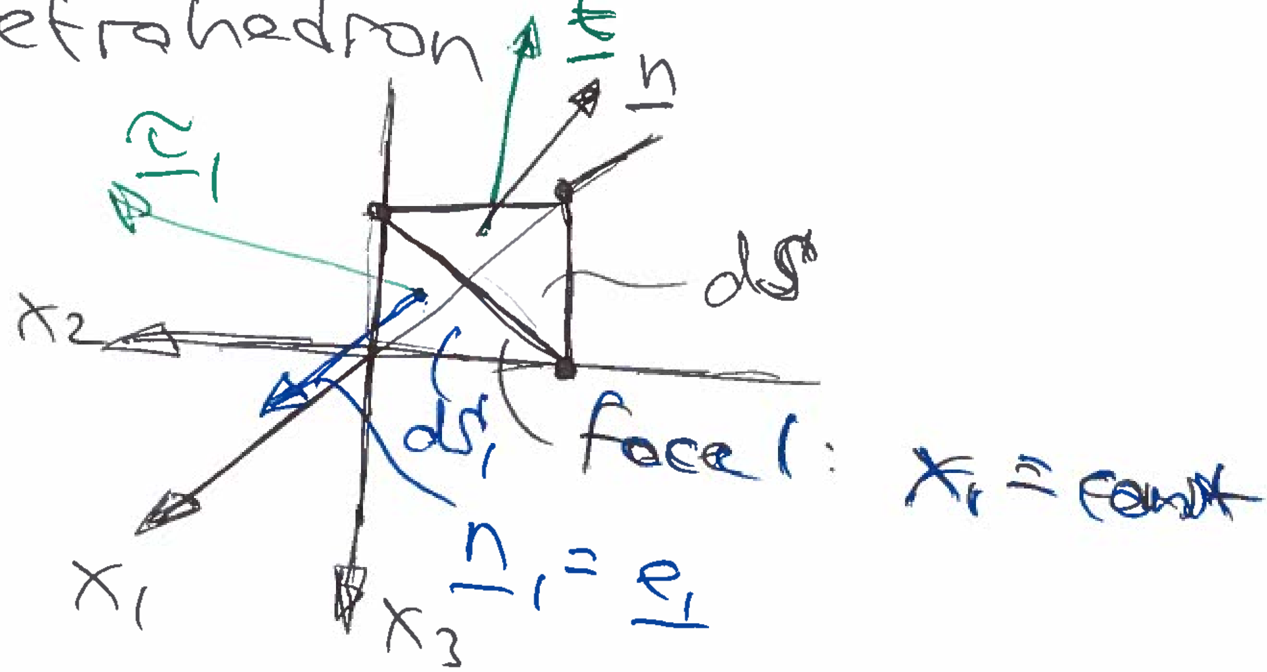
<u>Note:</u> Stress depends on:

- the posn. in fluid
- the orientation of the cut, i.e. $\underline{n}$

② The stress tensor

To examine dependence of $\underline{t}$ on $\underline{n}$:

Consider an infinitesimal tetrahedron



Face i is characterised by x_i = const & $\underline{n}_i = \underline{e}_i$.

<u>Trick:</u> Represent faces (orientation and area) by vectors which are parallel to $\underline{n}_i$ & have magnitude dS_i

<u>Then:</u> Sum of all area vectors on a closed body $= \underline{0}$

$$\boxed{\underline{n}_i \, dS_i + \underline{n} \, dS = \underline{0}}$$

(EXERCISE).

multiply by $\underline{n}_j = \underline{e}_j$

$$\underbrace{\underline{n}_i \cdot \underline{n}_j}_{\underbrace{\underline{e}_i \cdot \underline{e}_j}_{\delta_{ij}}} \, dS_i + \underbrace{\underline{n} \cdot \underline{e}_j}_{n_j} \, dS = 0$$

$$\boxed{dS_j = -n_j \, dS}$$

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<u>Now</u>: Balance of forces onto faces of tetrahedron:

$$\underbrace{\underline{t}\, dS}_{\substack{\text{force}\\\text{on}\\\text{general}\\\text{face}}} = - \underbrace{\underline{\tau}_j\, dS_j}_{\substack{\text{sum of forces}\\\text{on faces}\\1-3.}}$$

$$\underline{t}\, \cancel{dS} = \underline{\tau}_j n_j \cancel{dS}$$

In index notation

$$\boxed{t_i = \tau_{ij} n_j}$$

where τ_{ij} stress tensor.

τ_{ij} represents the stress/traction component in the positive i-direction on the face $x_j = \text{const.}$ whose outer unit normal points in the pos. x_j direction.