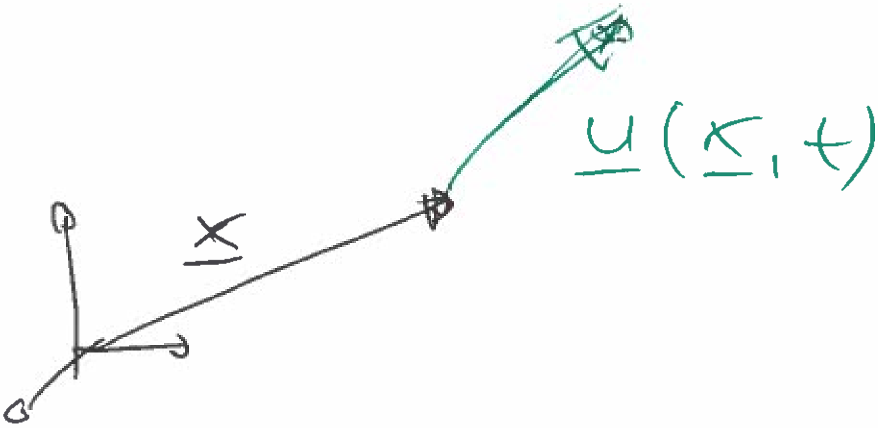
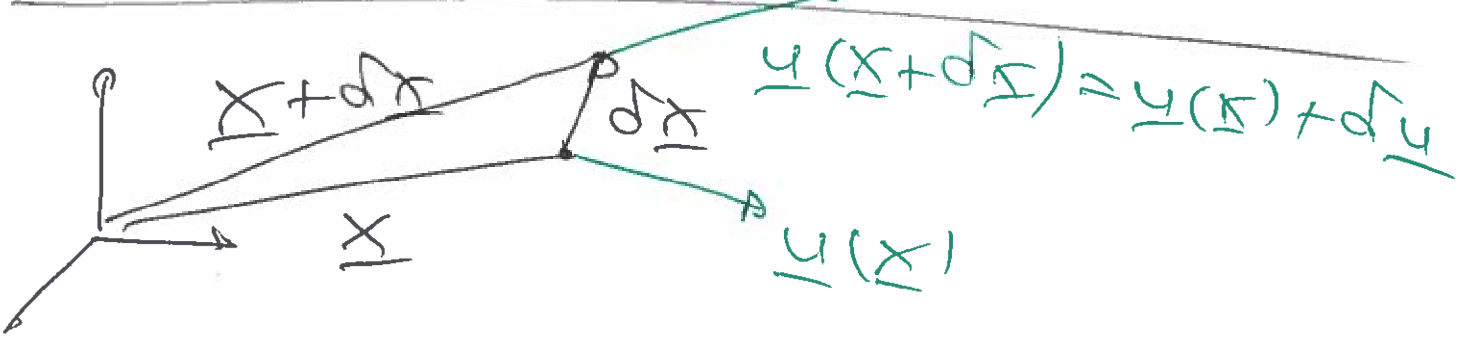


$$\frac{Du_i}{Dt} = \frac{\partial u_i}{\partial t} + \underbrace{u_j \frac{\partial u_i}{\partial x_j}}_{\text{"advection"}}$$

$\frac{Du_i}{Dt}$: accel experienced by particle

$\frac{\partial u_i}{\partial t}$: local (fixed x) rate of change of veloc $\rightarrow$ zero for "steady" flows

"advection" $\rightarrow$ zero if $\frac{\partial u_i}{\partial x_j} = 0$



$$\delta u_i = \frac{\partial u_i}{\partial x_j}\,\delta x_j$$

If $\frac{\partial u_i}{\partial x_j} = 0$ then $\delta u_i = 0$

$\Rightarrow$ translation; i.e. all fluid particles near P move with the same veloc.

$\Rightarrow$ $\frac{\partial u_i}{\partial x_j}$ must contain the other "modes".

To see this split $\frac{\partial u_i}{\partial x_j}$ into its sym. & anti-sym. part:

$$\frac{\partial u_i}{\partial x_j} = \underbrace{\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)}_{\varepsilon_{ij} = \varepsilon_{ji}} + \underbrace{\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right)}_{\omega_{ij} = -\omega_{ji}}$$

$$\delta u_i = \underbrace{\varepsilon_{ij}\,\delta x_j}_{\text{deformation}} + \underbrace{\omega_{ij}\,\delta x_j}_{\text{rotation}}$$

① Rigid body rotation / vorticity

Consider change in veloc. δu_i due to

$$\omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right) = -\omega_{ji}$$

$$\delta u_i = \omega_{ij}\,\delta x_j$$

in matrix form:

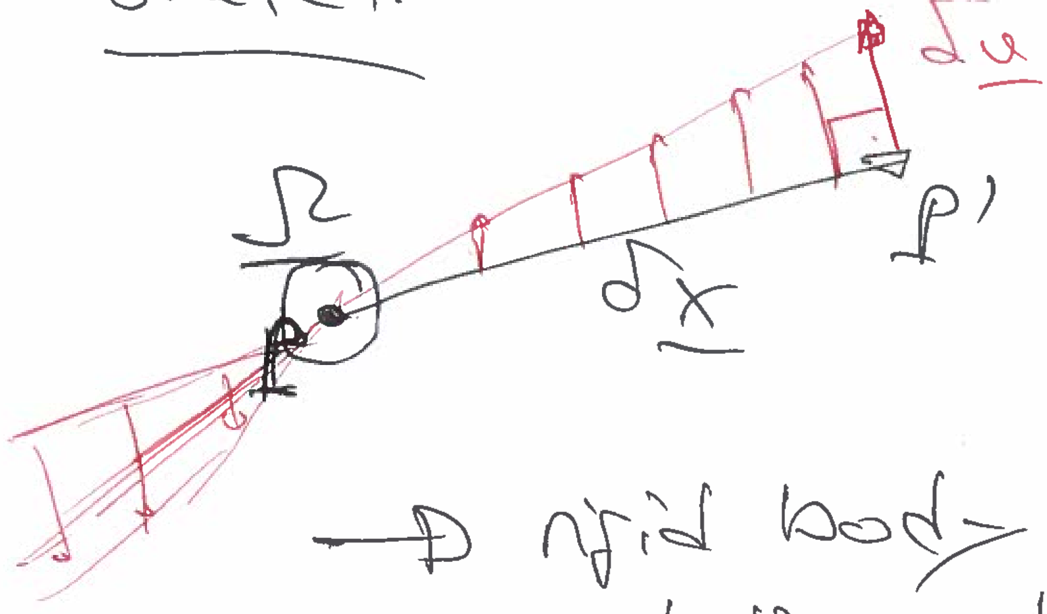
$$\begin{pmatrix} \delta u_1 \\ \delta u_2 \\ \delta u_3 \end{pmatrix} = \begin{pmatrix} 0 & -\Omega_3 & \Omega_2 \\ \Omega_3 & 0 & -\Omega_1 \\ -\Omega_2 & \Omega_1 & 0 \end{pmatrix} \begin{pmatrix} \delta x_1 \\ \delta x_2 \\ \delta x_3 \end{pmatrix}$$

can write this as

$$\delta \underline{u} = \underline{\Omega} \times \delta \underline{x}$$

$$\underline{\Omega} = \begin{pmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{pmatrix} = \text{rate of rotation vector.}$$

Sketch:



$\rightarrow$ rigid body rotation about P

$\underline{\delta u} = \underline{\Omega} \times \underline{\delta x}$ is the velocity induced at P' by a rigid body rotation with angular velocity $\underline{\Omega}$ about P.

$$\underline{\Omega} = \frac{1}{2} \underbrace{\nabla \times \underline{u}}$$

$\underline{\omega}$ = vorticity.

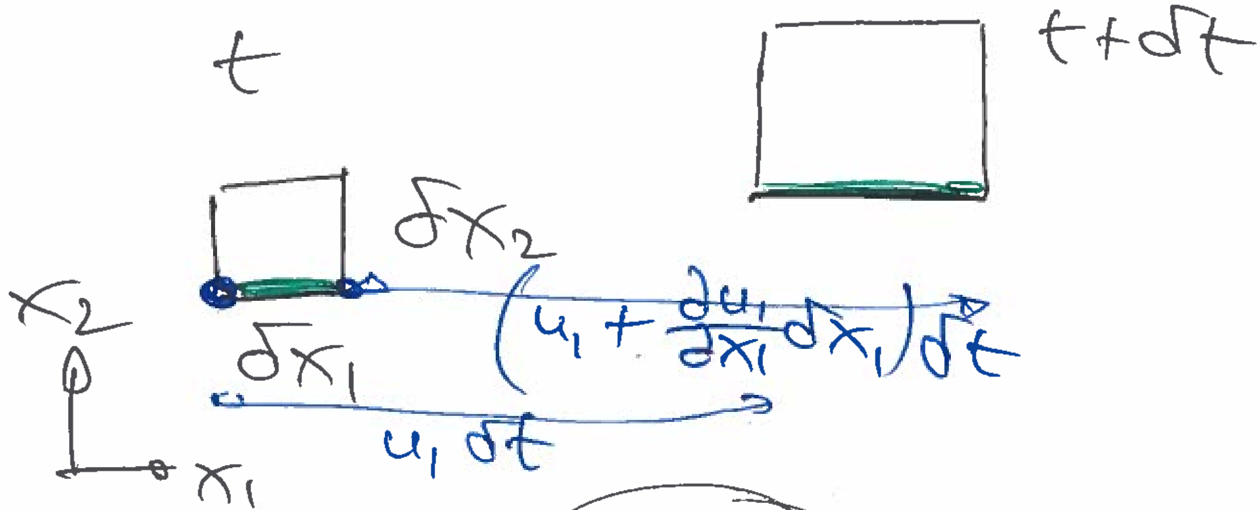
②.1 The rate of strain

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) = \varepsilon_{ji}$$

contains shearing & dilatation / stretching.

(i) Extensional rate of strain

Illustration: (2D)



$$\text{strain} = \frac{\text{length} - \text{old length}}{\underbrace{\text{old length}}_{\delta x_1}}$$

$$= \frac{\{\cancel{\delta x_1} + (\cancel{u_1} + \frac{\partial u_1}{\partial x_1}\cancel{\delta x_1})\delta t - \cancel{u_1 \delta t}\} - \cancel{\delta x_1}}{\cancel{\delta x_1}}$$

6

$$\text{strain} = \frac{\partial u_1}{\partial x_1} \delta t$$

rate of strain $\dfrac{\partial\, \text{strain}}{\partial t}$

$$= \frac{\partial u_1}{\partial x_1} = \varepsilon_{11}$$

Similar for other directions.

ε_{11} etc (the diagonal entries of the rate of strain tensor) represent the extensional rate of strain in the direction of the coordinate axes.