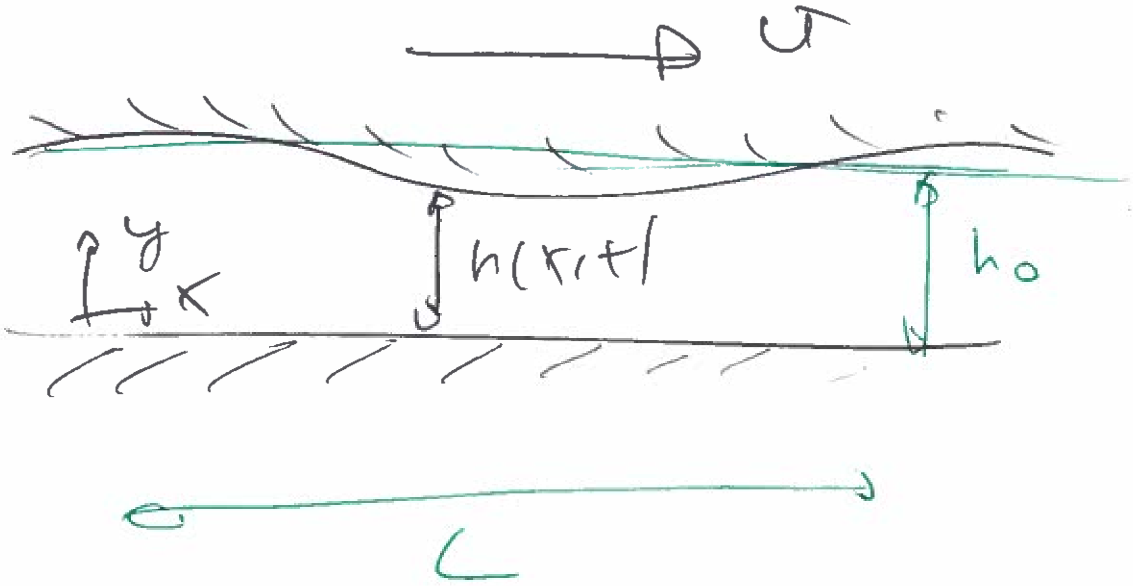


$\frac{h_0}{L} \ll 1$:

$$\boxed{\begin{aligned} 0 &= -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} \\ 0 &= -\frac{\partial p}{\partial y} \end{aligned}} \begin{aligned} &+ O\left(\frac{h_0}{L}\right) \\ &+ O\left(\frac{h_0}{L}\right) \end{aligned}$$

parallel flow eqns

$$\boxed{u(x,y,t) = \frac{1}{2\mu}\frac{\partial p}{\partial x}\left(y^2 - h(x,t)\,y\right) + U\frac{y}{h(x,t)}} \quad (*)$$

$$\boxed{Q(x,t) = \int_0^{h(x,t)} u(x,y,t)\,dy}$$

$$\boxed{\frac{\partial Q}{\partial x} = -\frac{\partial h}{\partial t}}$$

$$Q = -\frac{1}{12\mu}\frac{\partial p}{\partial x}h^3 + \frac{1}{2}Uh$$

into $\frac{\partial Q}{\partial x} = -\frac{\partial h}{\partial t}$:

$$\frac{\partial Q}{\partial x} = \boxed{-\frac{1}{12\mu}\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right) + \frac{1}{2}U\frac{\partial h}{\partial x} = -\frac{\partial h}{\partial t}}$$

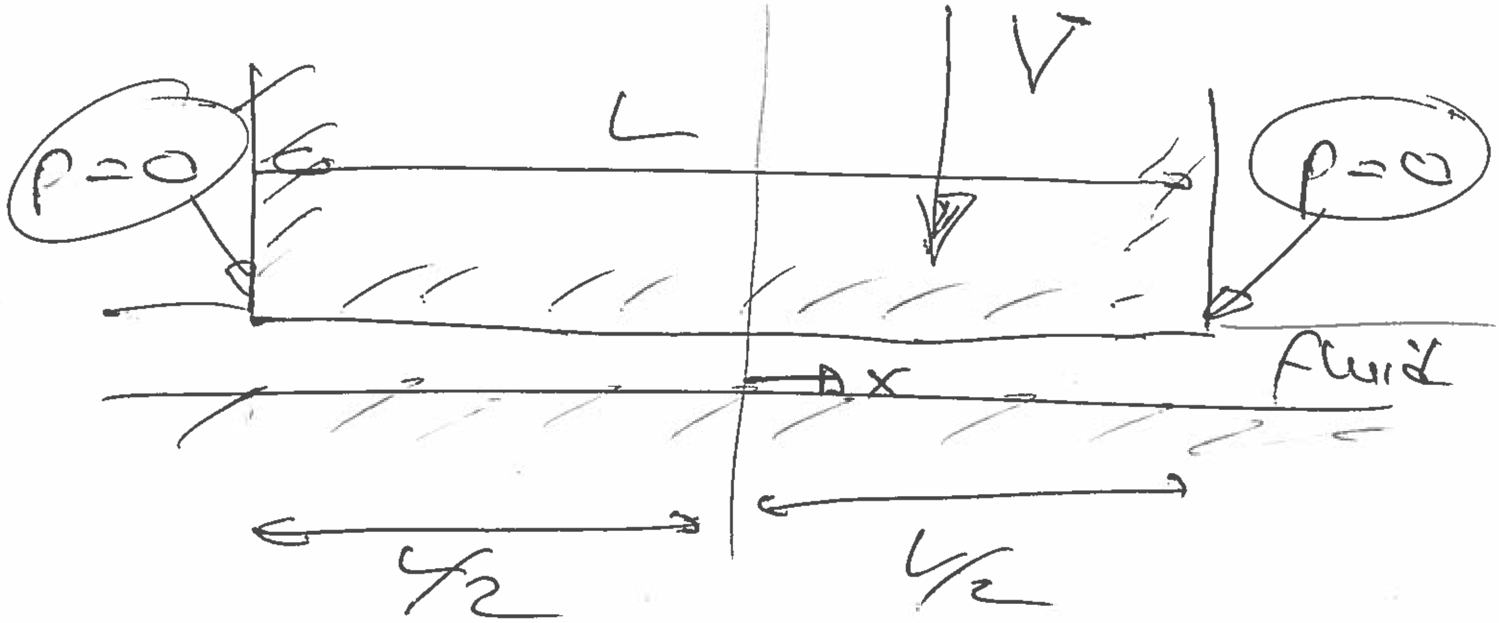
$$\boxed{\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right) = 12\frac{\partial h}{\partial t} + 6U\frac{\partial h}{\partial x}}$$

Reynolds' lubrication eqn.

For given U & $h(x,t)$ this PDE for $p(x,t)$. Once we have the pressure we obtain $u(x,y,t)$ from (*)

Example:

Squeeze film



Here: $U = 0$

$$h(x,t) = h_0 - Vt$$

assume $\frac{h_0}{L} \ll 1$

$$\frac{\partial h}{\partial x} = 0$$

so lubrication eqn:

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right) = 12\dot{h}$$

$$\frac{h^3}{\mu}\frac{\partial p}{\partial x} = 12\dot{h}x + \tilde{A}(t)$$

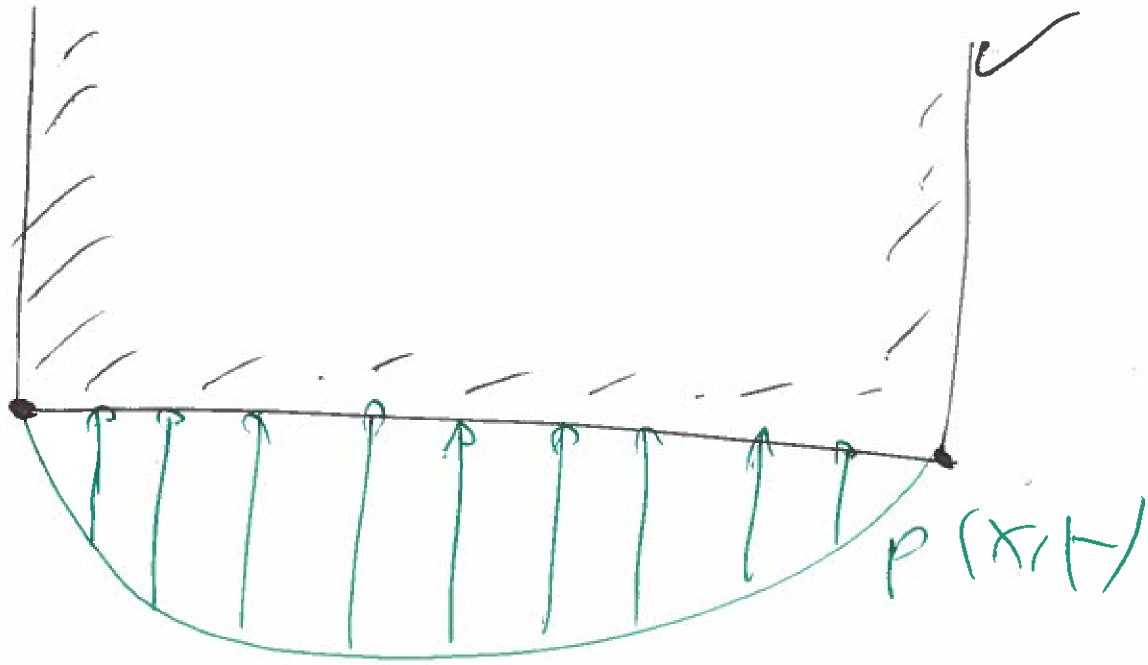
$$\frac{\partial p}{\partial x} = \frac{12 \dot{h} \mu}{h^3} x + A(t)$$

$$p(x) = \frac{6 \dot{h} \mu}{h^3} x^2 + A(t) x + B(t)$$

~~BCs~~ <u>BCs</u>: $p(x = -\frac{L}{2}, t) =$

$p(x = \frac{L}{2}, t) = 0$

$$\boxed{p(x,t) = \frac{6 \dot{h} \mu}{h^3}\left(x^2 - \left(\frac{L}{2}\right)^2\right)}$$



$h \to 0$ as t increases

when $t = \frac{h_0}{V}$: $h = 0$

(8)

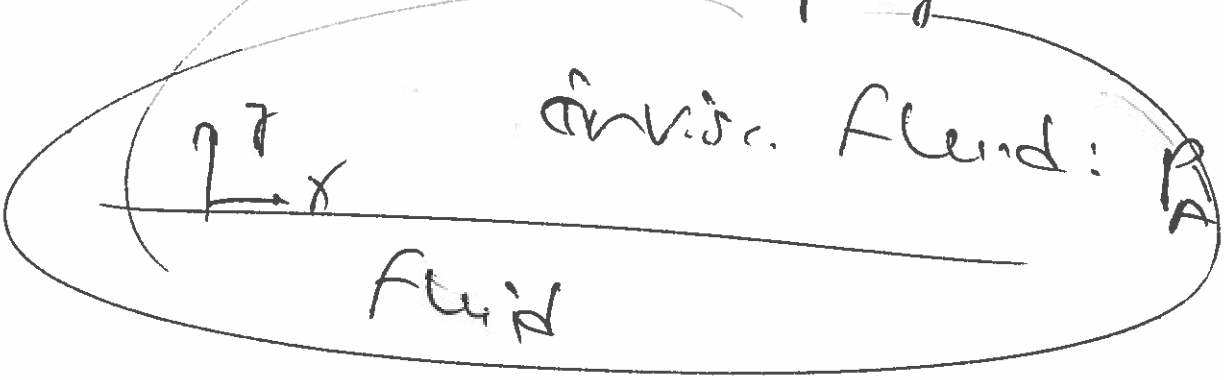
As $t \to \frac{h_0}{V}$: $p(x,t) \to \infty$

This is the essence of lubrication. Huge (infinite) forces would be required to bring the two plates into contact.

CARTESIAN

- kinematics $\frac{Du_i}{Dt}$, ε_{ij}, ω_{ij} & interpretation
- Cauchy, const. eqn, N-St
- BC: no slip, traction BC

$t_i = \tau_{ij} n_j$



$p = p_A \qquad \frac{\partial u}{\partial y} = 0$

- parallel flow
- primitive var. problems u, v, p
- nondim (N-St); linearity & dimensionality args, non-unif. of scal.

- Stream fct. for Stokes

$$\nabla^4 \psi = 0$$

$$u = \frac{\partial \psi}{\partial y}$$

$$v = -\frac{\partial \psi}{\partial x}$$

u, v BCs $\rightarrow$ BCs for ψ

---

show that : solve

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