~~§2~~ Fluid mechanics

3 steps: I Describe (mathematically) the flow field / motion of fluid particles. (Kinematics)

II Formulate the equations of motion (balance of forces acting on fluid particles); stresses

III Constitutive eqn.: relate Kinematics & stresses.

} Navier Stokes eqns.

Then lots of examples.

§2 Kinematics

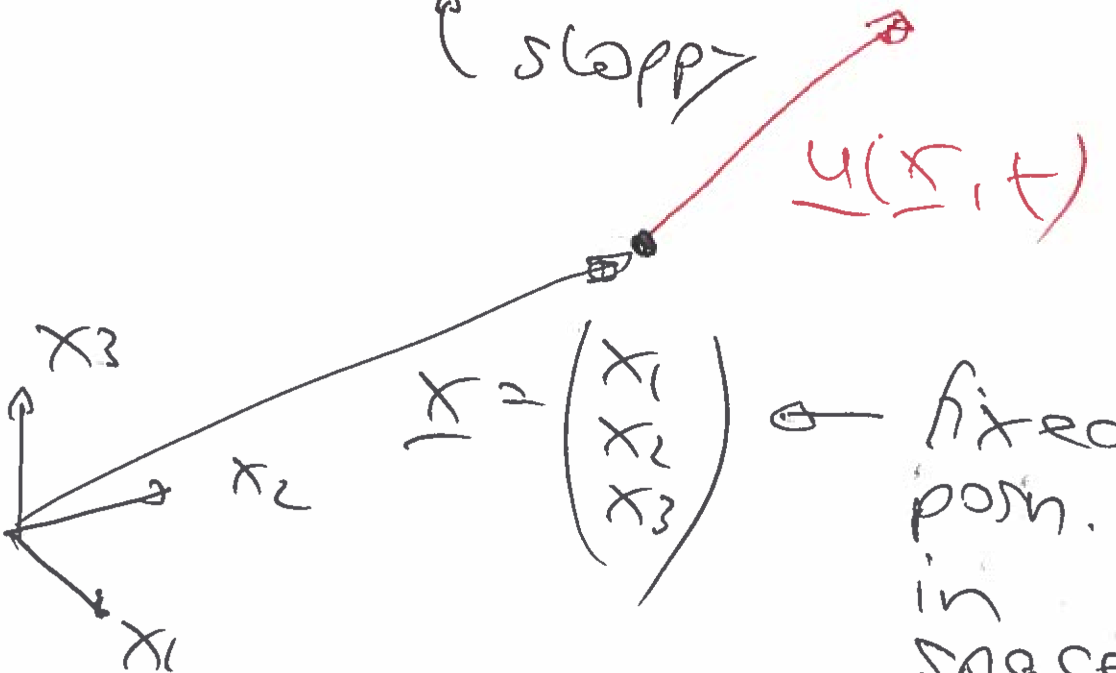
The Eulerian flow field

Assume we know the velocity $\underline{u}$ as a fct of the 3 spatial coordinates $(x_1, x_2, x_3) = (x, y, z)$ and time t.

$$\underline{u} = \underline{u}(x_1, x_2, x_3, t) = \underline{u}(\underline{x}, t)$$

$$u_i = u_i(x_j, t)$$

(sloppy)



$\underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ ← fixed posn. in space.

At time t the material particle of position $\underline{x}$ has velocity $\underline{u}$.
(Eulerian description).

This has important implications

E.g.: Acceleration of fluid particles:

The material derivative

The position $\underline{x}$ of a particle is given by

$$\underline{x} = \underline{x}^P(t) = \begin{pmatrix} x_1^P(t) \\ x_2^P(t) \\ x_3^P(t) \end{pmatrix}$$

Velocity of that particle that is currently (at time t) at position $\underline{x}^P(t)$

$$\underline{u}(\underline{x}^P(t), t)$$

$$\underline{u}(x_1^p(t), x_2^p(t), x_3^p(t), t)$$

So the accel. of the particle is the rate of change of that velocity w.r.t. time t.

$$\frac{d\underline{u}}{dt} = \frac{\partial \underline{u}}{\partial t} + \frac{\partial \underline{u}}{\partial x_1}\frac{\partial x_1^p}{\partial t} + \frac{\partial \underline{u}}{\partial x_2}\frac{\partial x_2^p}{\partial t} + \frac{\partial \underline{u}}{\partial x_3}\frac{\partial x_3^p}{\partial t}$$

$$\frac{du_i}{dt} = \frac{\partial u_i}{\partial t} + \frac{\partial u_i}{\partial x_j}\underbrace{\frac{\partial x_j^p}{\partial t}}_{u_j}$$

$$\boxed{\frac{du_i}{dt} = \frac{\partial u_i}{\partial t} + u_j\frac{\partial u_i}{\partial x_j}}$$

in symbolic form:

$$\boxed{\frac{d\underline{u}}{dt} = \frac{\partial \underline{u}}{\partial t} + \underline{u} \cdot \nabla \underline{u}}$$

often written as

$$\frac{D\underline{u}}{Dt}$$

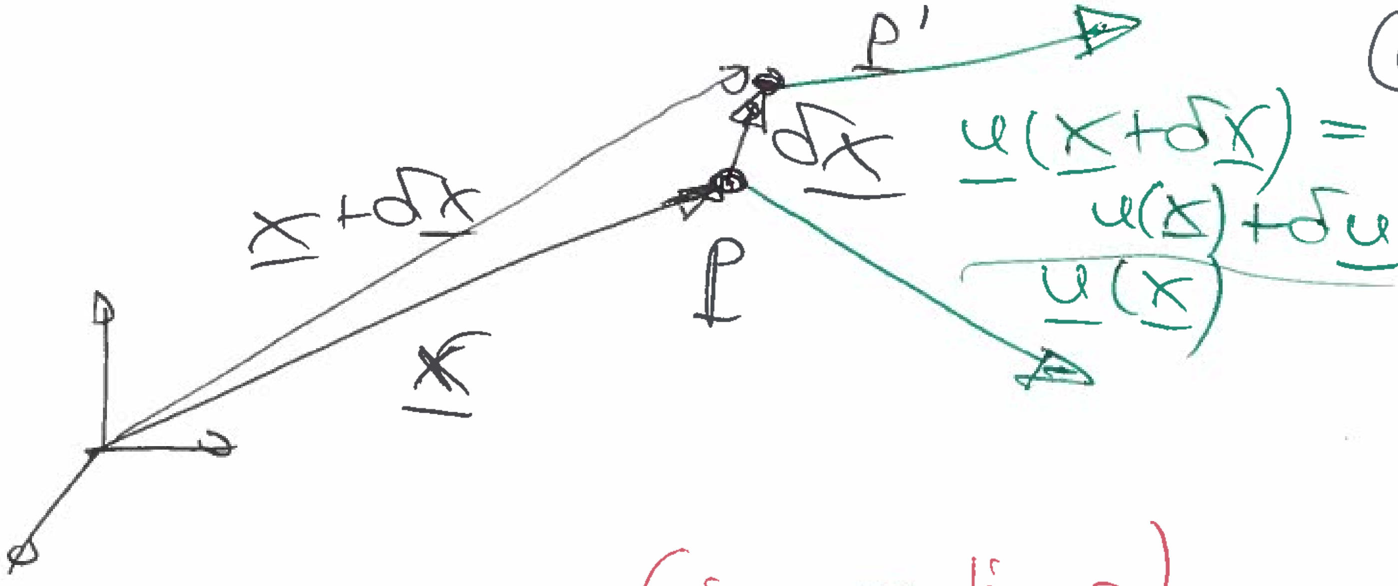
to highlight that this follows a particle.

The rate of strain tensor & the vorticity

The velocity field contains:

- translation
- rotation
- shearing
- dilation

How do we identify these?



(ignore time)

i.e. the position changes from

$$\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} \text{ to } \begin{pmatrix}x_1+\delta x_1\\x_2+\delta x_2\\x_3+\delta x_3\end{pmatrix}$$

veloc. of P' is:

$$\underline{u}(x_1+\delta x_1, x_2+\delta x_2, x_3+\delta x_3) =$$

$$= \underline{u}(x_1,x_2,x_3) + \frac{\partial \underline{u}}{\partial x_1}\delta x_1 + \frac{\partial \underline{u}}{\partial x_2}\delta x_2 + \frac{\partial \underline{u}}{\partial x_3}\delta x_3 + \dots$$

$$u_i(x_k+\delta x_k) = u_i(x_k) + \left.\frac{\partial u_i}{\partial x_j}\right|_{P} \delta x_j + \dots$$

$\delta x_j \to 0$

$$du_i = \underbrace{\left.\frac{\partial u_i}{\partial x_j}\right|_{P}}_{\text{velocity gradient tensor}} dx_j$$

3x3 matrix