~~Bla bla~~ bla ~~bla~~!

1. Index notation

various ways of expressing a vector

$$\underline{a} = a_1 \underline{e_1} + a_2 \underline{e_2} + a_3 \underline{e_3} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

symbolic | in components relative to some basis $(\underline{e_1}, \underline{e_2}, \underline{e_3}) = (\underline{i}, \underline{j}, \underline{k})$ | in comp.

Conventions:

Simply write down one generic term of each vector (eqn.)

$$\underline{c} = \underline{a} + \underline{b} \longrightarrow c_i = a_i + b_i$$

i is a free index which takes values 1, 2, 3

Example:

$$\nabla\phi = \begin{pmatrix} \frac{\partial\phi}{\partial x_1} \\ \frac{\partial\phi}{\partial x_2} \\ \frac{\partial\phi}{\partial x_3} \end{pmatrix} \rightarrow \frac{\partial\phi}{\partial x_i}$$

Convention 2:

Summation convention.

Rule: automatically sum over repeated (dummy) indices.

Example:

$$\underline{u}\cdot\underline{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$$

$$= \cancel{\sum_{i=1}^{3}} \underbrace{u_i v_i} = \underbrace{u_i v_i} = \underbrace{u_j v_j}$$

Example:

$$\operatorname{div}\underline{u} = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = \frac{\partial u_j}{\partial x_j}$$

Higher-order "tensors"

So far: index notation to represent components of vectors (one free index)

Higher-order "tensors" arise naturally in many applications:

Example:

$$\underline{\sigma} = \underset{\sim}{T} \cdot \underline{n}$$

vector

stress tensor (3x3 matrix)

vector

$$\sigma_i = T_{ij} n_j = T_{ik} n_k$$

Note: free index (i) has to appear on both sides.

One special second order tensor is the Kronecker delta:

$$\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

$$\begin{array}{cc} & j \rightarrow \\ i \downarrow & \left[\delta_{ij}\right] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{array}$$

δ_{ij} has an interesting property in a summation:

$$b_j = \boxed{a_i \delta_{ij} = \sum_{i=1}^{3} \underbrace{a_i \delta_{ij}}_{\text{only } a_j \text{ "survives"}} = a_j}$$

"δ_{ij} exchanges indices"

<u>Disclaimer:</u> Not every object with subscripts is a tensor.