

MATH35001: EXAMPLE SHEET¹ III

- 1.) A 2D flow field is given by $u_1 = 1 + x_2^2 \cos x_1$ and $u_2 = 3 + Ax_2^3 \sin x_1$. For which value of the constant A is this flow field consistent with the assumption of incompressibility?
- 2.) The observation of a 2D flow field shows that the velocities along the four edges of the unit square ($x_i \in [0, 1]$) are given by

$$\begin{aligned} \mathbf{u} &= (3 + x_2, 4) \quad \text{on } x_1 = 0, \\ \mathbf{u} &= (3 + x_2 + x_2^3, 7 - \frac{1}{2}x_2^4) \quad \text{on } x_1 = 1, \\ \mathbf{u} &= (3, 4 + 3x_1^2) \quad \text{on } x_2 = 0, \\ \mathbf{u} &= (4 + x_1^2, 4 - \frac{1}{2}x_1 + 3x_1^2) \quad \text{on } x_2 = 1. \end{aligned}$$

Is the fluid incompressible? (Think carefully about this!).

- 3.) Consider the infinitesimal tetrahedron used in the derivation of the stress tensor and show that

$$\mathbf{n}_i ds_i + \mathbf{n} ds = 0$$

where the \mathbf{n}_i are the outside unit normal vectors on the three faces on which $x_i = \text{const.}$, the ds_i are their areas, \mathbf{n} is the outside unit normal vector on the fourth (general) face and ds is its area. [Hint: Express the areas via cross products of the three vectors forming the tetrahedron's edges on the coordinate axes].

Coursework

Please exchange your solution to questions 1 and 2 with your "marking buddy" and assess each other's work, using the master solution made available on the course webpage (probably in week 4).

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