Cauchy's eqn:

$$\frac{\partial \tau_{ij}}{\partial x_j} + \rho f_i = \rho \frac{Du_i}{Dt}$$

$$= \rho \left(\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right)$$

Stress tensor

Body force (per unit mass)

density

Accel.

4. Symmetry of stress tensor

$$\boxed{\tau_{ij} = \tau_{ji}}$$

Constitutive eqns

provide a link between the kinematics of the flow & τ_{ij}.

Restriction: we will only consider incompressible fluids.

Observation:

Fluids:

- can generate hydrostatic pressures.
- have a resistance to shear flows (viscosity!) knife & honey.
- do not generate internal stresses when subjected to rigid body motions.

$\Rightarrow \tau_{ij}$ should contain a hydrostatic pressure & depend on ε_{ij}.

A wide range of fluids (Newtonian fluids) behave:

$$\tau_{ij} = -p\,\delta_{ij} + 2\mu\,\varepsilon_{ij}$$

μ dynamic viscosity (has to be measured)

$$\tau_{ij} = -p\,\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Into Cauchy's eqn:

$$\rho\frac{Du_i}{Dt} = \rho f_i + \frac{\partial \tau_{ij}}{\partial x_j}$$

$$= \rho f_i + \frac{\partial}{\partial x_j}\left(-p\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right)$$

$$\rho \frac{Du_i}{Dt} = \rho f_i - \frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 u_i}{\partial x_j^2} + \mu \underbrace{\cancel{\frac{\partial}{\partial x_j}\left(\frac{\partial u_j}{\partial x_i}\right)}}$$

$$\underbrace{\cancel{\frac{\partial}{\partial x_i}\left(\frac{\partial u_j}{\partial x_j}\right)}}$$

$$\cancel{\mathrm{div}\, \underline{u} = 0}$$

$$\boxed{\rho\left(\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j}\right) = \rho f_i - \frac{\partial p}{\partial x_i} + \mu \underbrace{\frac{\partial^2 u_i}{\partial x_j^2}}_{\nabla^2 u_i}}$$

+ eqn of continuity

$$\boxed{\frac{\partial u_j}{\partial x_j} = 0}$$

Navier Stokes eqns.

A system four coupled nonlinear 2^{nd} order PDE. for 3 veloc. comp. & pressure.

Boundary & initial conditions