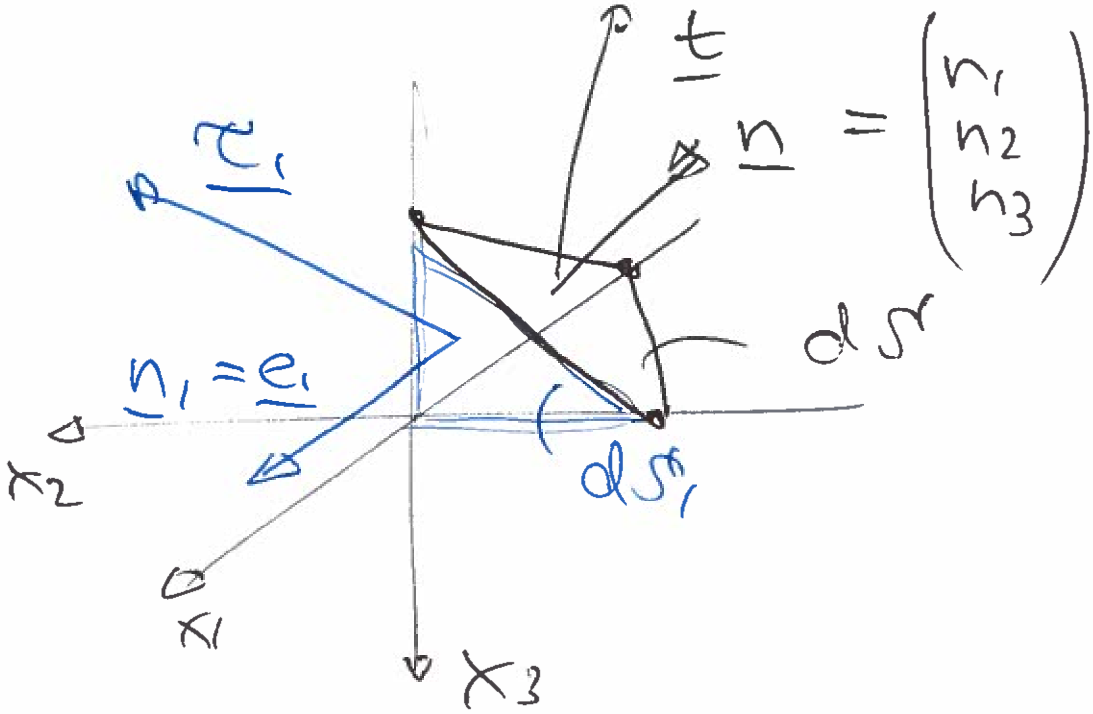


Geometry: $\boxed{dS_j = -n_j \, dS}$

Now: balance of forces $\stackrel{!}{=} \underline{0}$

$$\underline{t}\, dS = -\underline{\tau}_j \, dS_j$$

$$\underline{t}\, \cancel{dS} = \underline{\tau}_j \, n_j \, \cancel{dS}$$

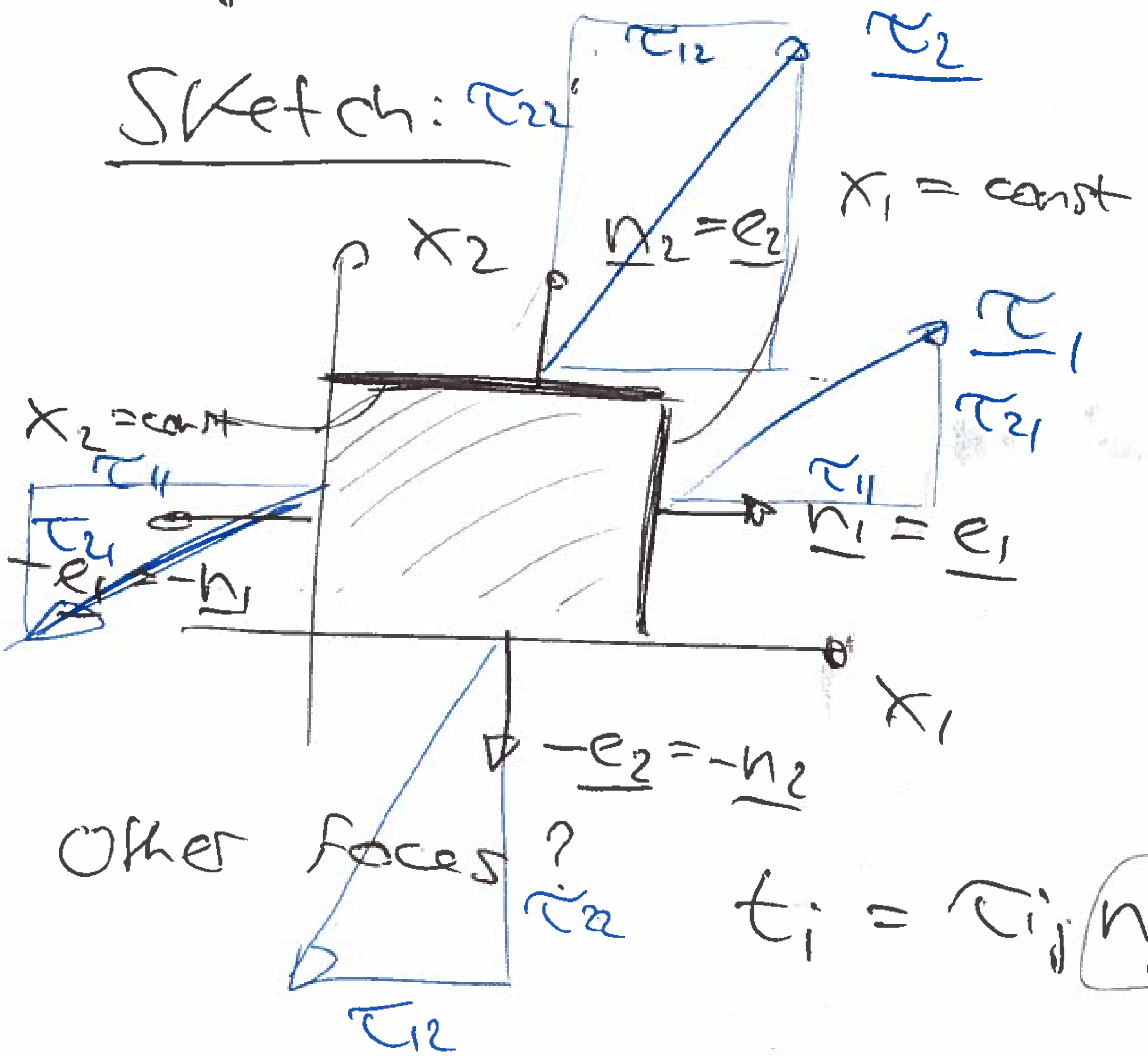
index notation:

$$\boxed{t_i = \tau_{ij} n_j}$$

τ_{ij} is the stress tensor

where τ_{ij} is the traction in the positive i-direction on the face where $x_j = \text{const.}$ whose outer unit normal points in the positive x_j-direction.

Sketch:



Other faces?

$$t_i = \tau_{ij} n_j$$

Later: we will have to establish what determines τ_{ij}

Particular stress states

(i) Hydrostatic pressure

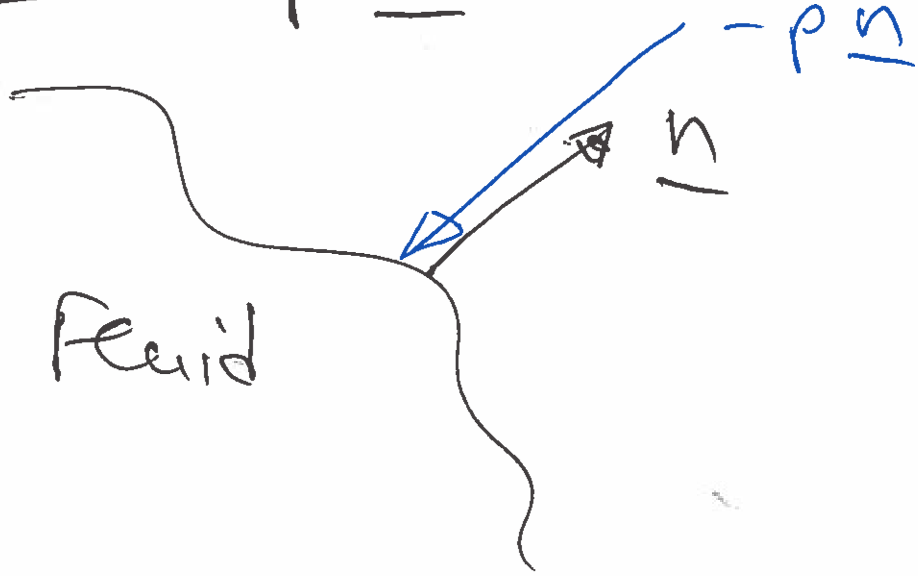
$$\tau_{ij} = -p\,\delta_{ij}$$

implies that traction is always normal to the plane & uniform in all directions, because

$$t_i = \tau_{ij} n_j$$

$$t_i = -p\,\delta_{ij} n_j = -p\,n_i$$

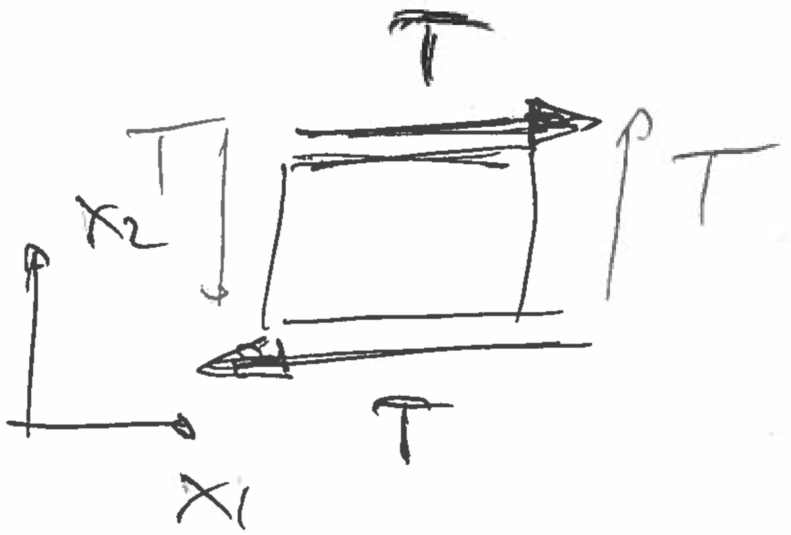
$$\underline{t} = -p\,\underline{n}$$



(ii) pure shear (2D) (4

e.g. $\tau_{12} = \tau_{21} = T$

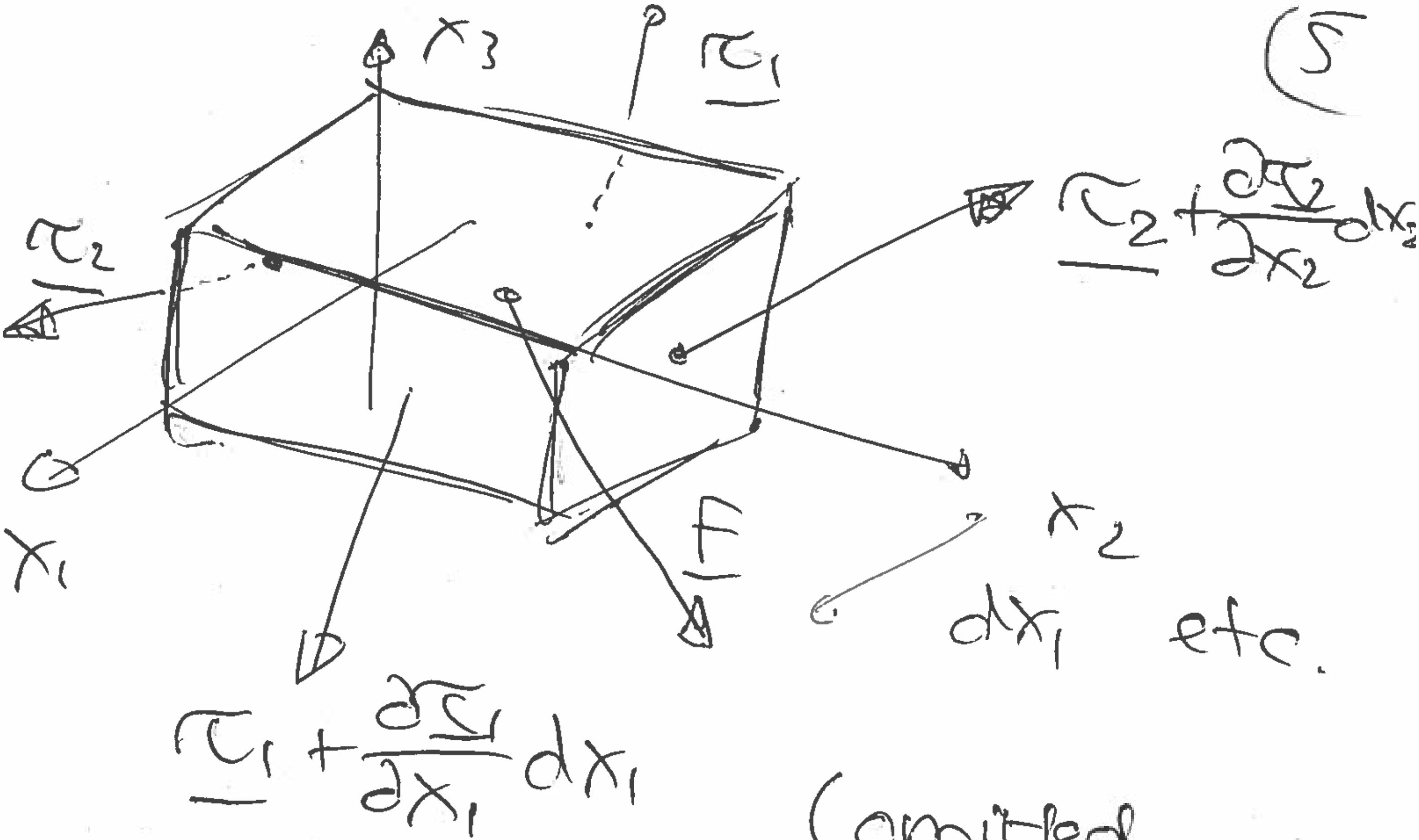
$\tau_{11} = \tau_{22} = 0$



3. Equilibrium of forces / Cauchy's eqn

Invoke Newton's law for an infinitesimal blob of fluid

$$\sum \text{forces} = \text{mass} \times \text{accel.}$$



(omitted x_3 faces)

Const terms from traction on opposite faces cancel leaving only the increments:

$$\frac{\partial \underline{\tau}_1}{\partial x_1}\cancel{dx_1 dx_2 dx_3} +$$

$$\frac{\partial \underline{\tau}_2}{\partial x_2}\cancel{dx_2 dx_1 dx_3} +$$

$$\frac{\partial \underline{\tau}_3}{\partial x_3}\cancel{dx_3 dx_1 dx_2} +$$

$$\rho \underline{F}\,\cancel{dx_1 dx_2 d_3} = \rho \frac{D\underline{u}}{Dt}\cancel{dx_1 dx_2 dx_3}$$

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Here: $\underline{F}$ = body force
(force per unit mass, e.g. gravity $\underline{F} = \underline{g}$)

Rewrite in components

$$\boxed{\frac{\partial \sigma_{ij}}{\partial x_j} + \rho F_i = \rho \frac{D u_i}{D t}}$$

Cauchy's eqn.