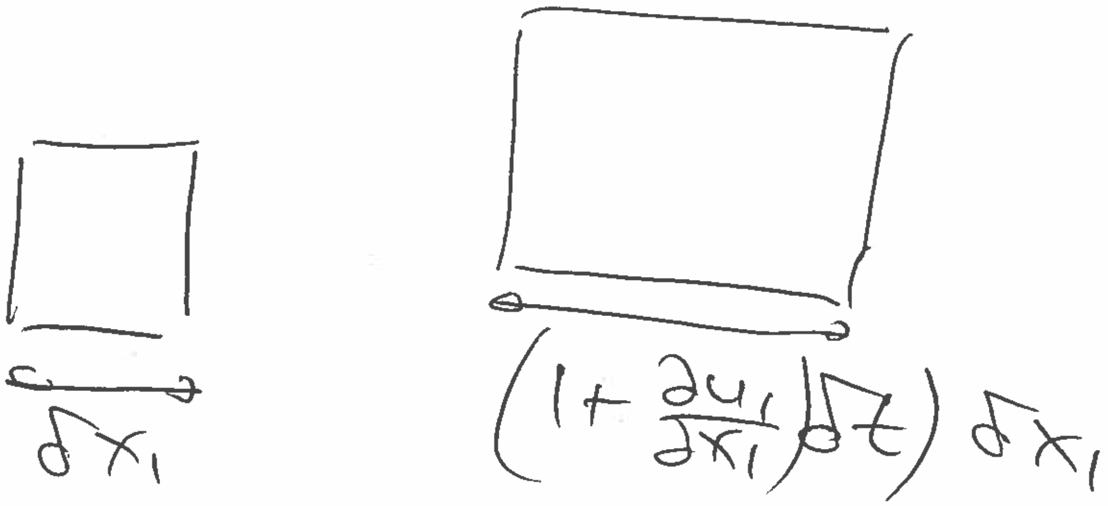
$\underline{u}$ "contains"

- translation ✓
- rotation ✓
- dilation ✓
- shear

$$\delta u_i = \frac{\partial u_i}{\partial x_j}\,\delta x_j$$

$$= \left[\underbrace{\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)}_{\varepsilon_{ij}=\varepsilon_{ji}} + \underbrace{\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}-\frac{\partial u_j}{\partial x_i}\right)}_{\omega_{ij}=-\omega_{ji}}\right]\delta x_j$$

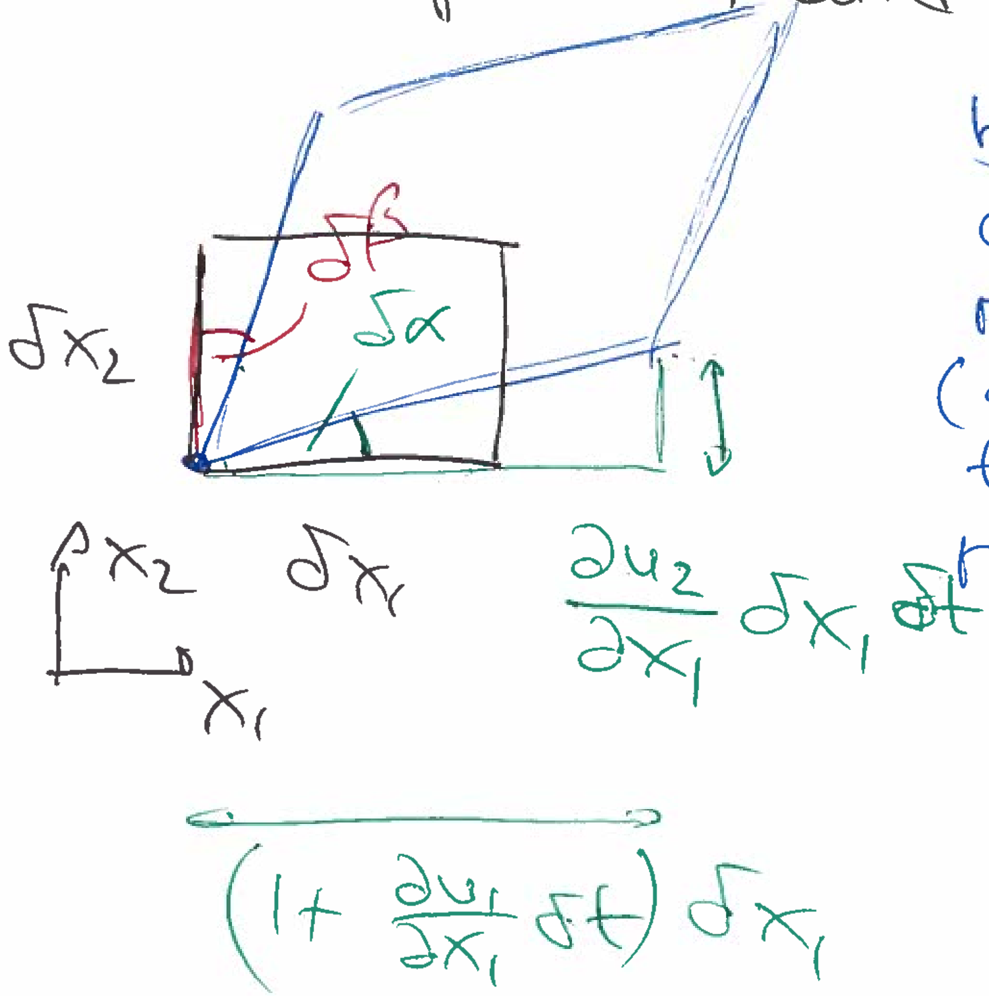



$\varepsilon_{11}$ extensional rate of $\underbrace{\text{strain}}_{\frac{\Delta \ell}{\ell}}$ of line elements in $x_1$-direction

## (ii) Shear rate of strain

Illustration in 2D

shear = change in shape (angles) of a small initially rectangular fluid element




bottom left corner realigned (undo translation & rotation)

See last lecture or sheet 1

$$\underbrace{\tan\delta\alpha}_{\to\,\delta\alpha} = \frac{\frac{\partial u_2}{\partial x_1}\cancel{\delta x_1}\,\delta t}{\left(1+\cancel{\frac{\partial u_1}{\partial x_1}\delta t}\right)\cancel{\delta x_1}}$$

as $\delta t \to 0$

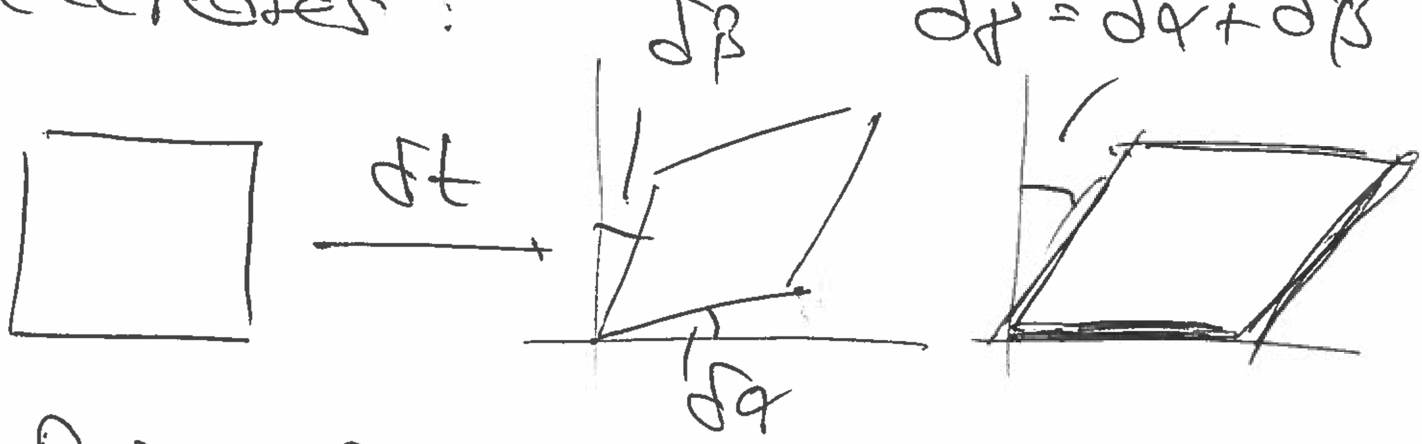
$$\frac{\partial \alpha}{\partial t} = \frac{\partial u_2}{\partial x_1}$$

$$\frac{D\alpha}{Dt} = \frac{\partial u_2}{\partial x_1}$$

Similarly

$$\frac{D\beta}{Dt} = \frac{\partial u_1}{\partial x_2}$$

Now consider the shear rate = the rate at which the initially right angle between two horizontal & vertical line elements decreases :




$$\frac{D\gamma}{Dt} = \frac{D\alpha}{Dt} + \frac{D\beta}{Dt} = \frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2} = 2\varepsilon_{12}$$

(Similar for other directions)

The off-diagonal entries of $\varepsilon_{ij}$ represent half the rate of strain in the $x_i$-$x_j$-plane.

<u>Summary:</u>

$$\underline{u}(\underline{x}+\delta\underline{x}) = \overbrace{\overbrace{\underline{u}(\underline{x})}^{\text{translation}} + \overbrace{\underline{\Omega}\times\delta\underline{x}}^{\text{rotation}}}^{\text{rigid body}} + \underbrace{\underset{\sim}{\varepsilon}\,\delta\underline{x}}_{\text{shear \& dilation}}$$

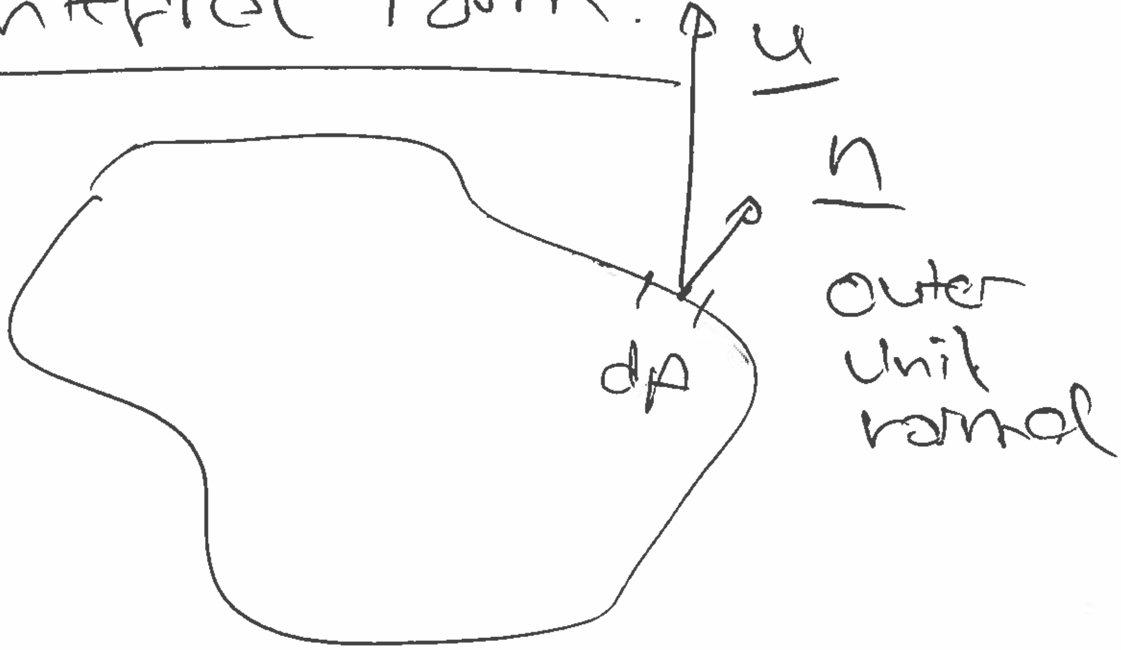
$$u_i(x_j+\delta x_j) = u_i(x_j) + \omega_{ij}\,\delta x_j + \varepsilon_{ij}\,\delta x_j$$

# Eqn. of continuity

**Physics:** "Mass is conserved".

Mass flux into a spatially fixed volume = rate of change of mass in this volume.

**Integral form:**




Mass flux:

density $\rho\left[\frac{kg}{m^3}\right]$ × velocity normal to face $\left[\frac{m}{sec}\right]$ × area $\left[m^2\right]$

= rate of change of mass $\left[\frac{kg}{sec}\right]$

$$\boxed{-\oint (\rho\,\underline{u})\cdot \underline{n}\, dA = \int \frac{d\rho}{dt}\, dV}$$

Diverfence theorem:

$$\int \operatorname{div} \underline{v}\, dV = \oint \underline{v}\cdot\underline{n}\, dA$$

use $\underline{v} = \rho\,\underline{u}$

$$\int \left[ \frac{d\rho}{dt} + \operatorname{div}(\rho\,\underline{u}) \right] dV = 0$$

This is ~~as~~ an integral over a fixed but arbitrary volume. If the integree is zero for any volume then

$$\frac{d\rho}{dt} + \operatorname{div}(\rho\,\underline{u}) = 0$$

pointwise.