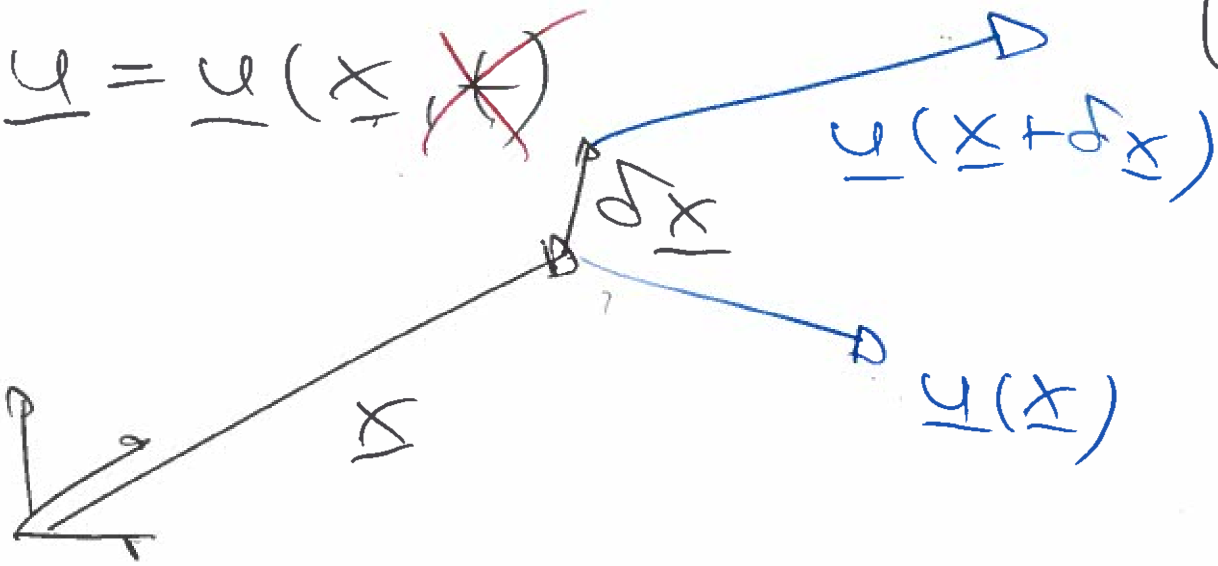
$\underline{u} = \underline{u}(\underline{x}, \cancel{t})$



$$\underline{u}(\underline{x}+\delta\underline{x}) = \underline{u}(\underline{x}) + \delta\underline{u}$$

$$\delta u_i = \underbrace{\frac{\partial u_i}{\partial x_j}}_{\text{veloc. gradient tensor}} \delta x_j + \dots$$

Translation $\rightarrow \underline{u} = \text{const} \rightarrow \frac{\partial u_i}{\partial x_j} = 0$
$\longrightarrow \delta u_i = 0$

[<u>Claim</u>: fluid motion "contains"

- translation
- rotation
- shearing
- dilation]

2

To see that $\frac{\partial u_i}{\partial x_j}$ "contains" rotation, shearing & dilation.

To see this: split $\frac{\partial u_i}{\partial x_j}$ into sym. & antisym. parts

$$\frac{\partial u_i}{\partial x_j} = \underbrace{\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)}_{\text{sym } \varepsilon_{ij} = \varepsilon_{ji}} + \underbrace{\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right)}_{\text{antisym. } \omega_{ij} = -\omega_{ji}}$$

ε_{ij} = rate of strain tensor

ω_{ij} = rate of rotation tensor

$$\delta u_i = \frac{\partial u_i}{\partial x_j} \delta x_j$$

$$\delta u_i = \underbrace{\varepsilon_{ij}\,\delta x_j}_{\text{shear \& dilation}} + \underbrace{\omega_{ij}\,\delta x_j}_{\text{rotation}}$$

① Rigid body rotation / vorticity

Consider the effect of 2^{nd} term on change in veloc:

$$\omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right) = -\omega_{ji}$$

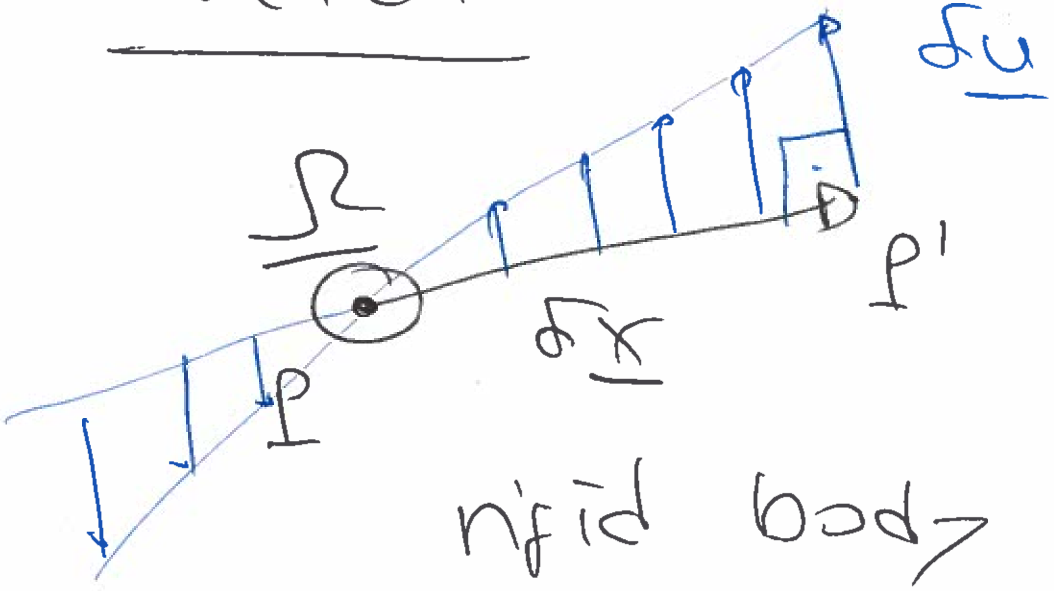
$$\delta u_i = \omega_{ij}\,\delta x_j$$

$$\begin{pmatrix} \delta u_1 \\ \delta u_2 \\ \delta u_3 \end{pmatrix} = \begin{pmatrix} 0 & -\Omega_3 & \Omega_2 \\ \Omega_3 & 0 & -\Omega_1 \\ -\Omega_2 & \Omega_1 & 0 \end{pmatrix} \begin{pmatrix} \delta x_1 \\ \delta x_2 \\ \delta x_3 \end{pmatrix}$$

$$\boxed{\delta \underline{u} = \underline{\Omega} \times \delta \underline{x}}$$

$$\underline{\Omega} = \begin{pmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{pmatrix}$$ rate of rotation vector.

Sketch:



rigid body rotation.

$$\underline{\Omega} = \frac{1}{2} \nabla \times \underline{u} = \frac{1}{2} \begin{pmatrix} \frac{\partial}{\partial x_1} \\ \frac{\partial}{\partial x_2} \\ \frac{\partial}{\partial x_3} \end{pmatrix} \times \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

$$\underline{\Omega} = \frac{1}{2} \begin{pmatrix} \frac{\partial u_3}{\partial x_2} - \frac{\partial u_2}{\partial x_3} \\ \frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1} \\ \frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2} \end{pmatrix} = \frac{1}{2} \underline{\omega}$$

vorticity vector.

(5

2. 1. The rate of strain

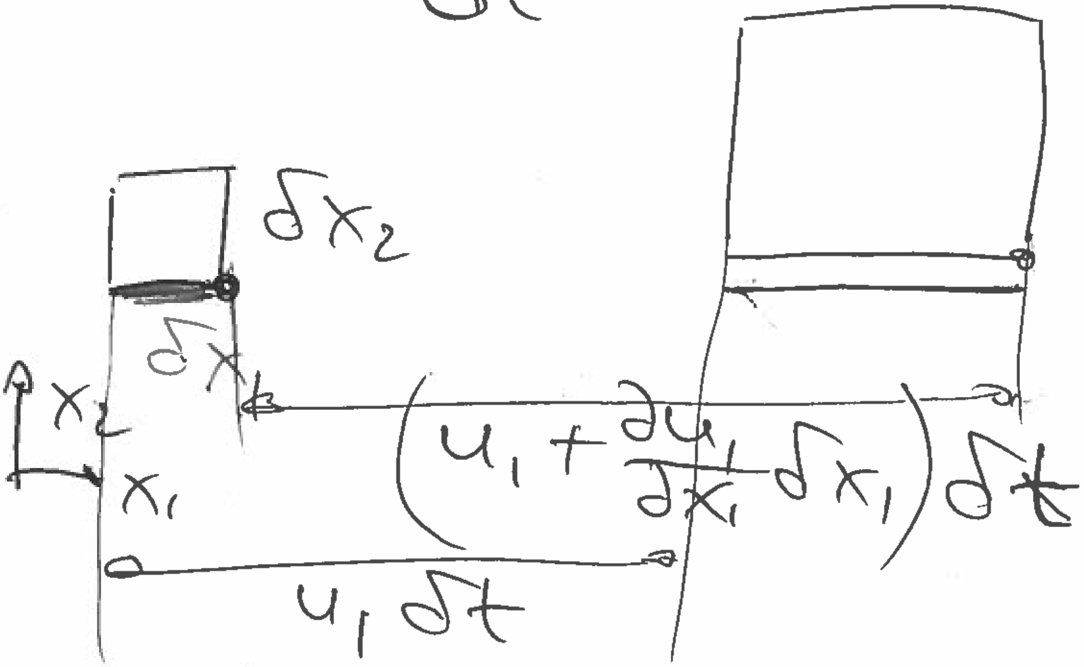
$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Claim: This "contains" shearing & dilatation

(i) Extensional rate of strain

Illustration (2D)

δt



$$\text{strain} = \frac{\text{length} - \text{old length}}{\text{old length}}$$

$$= \frac{\left(\cancel{\delta x_1} + \left(\cancel{u_1} + \frac{\partial u_1}{\partial x_1}\cancel{\delta x_1}\right)\delta t - \cancel{u_1 \delta t}\right) - \cancel{\delta x_1}}{\cancel{\delta x_1}}$$

$$= \frac{\partial u_1}{\partial x_1}\delta t$$

$$\text{rate of strain} = \frac{\partial\,\text{strain}}{\partial t}$$

Axial rate of strain in the x_1 direction

$$= \frac{\partial u_1}{\partial x_1} = \varepsilon_{11}$$

(similar for other directions)

ε_{11} etc (diagonal entries of rate of strain tensor) represent the extensional rate of strain in the direction of the x_1-axis