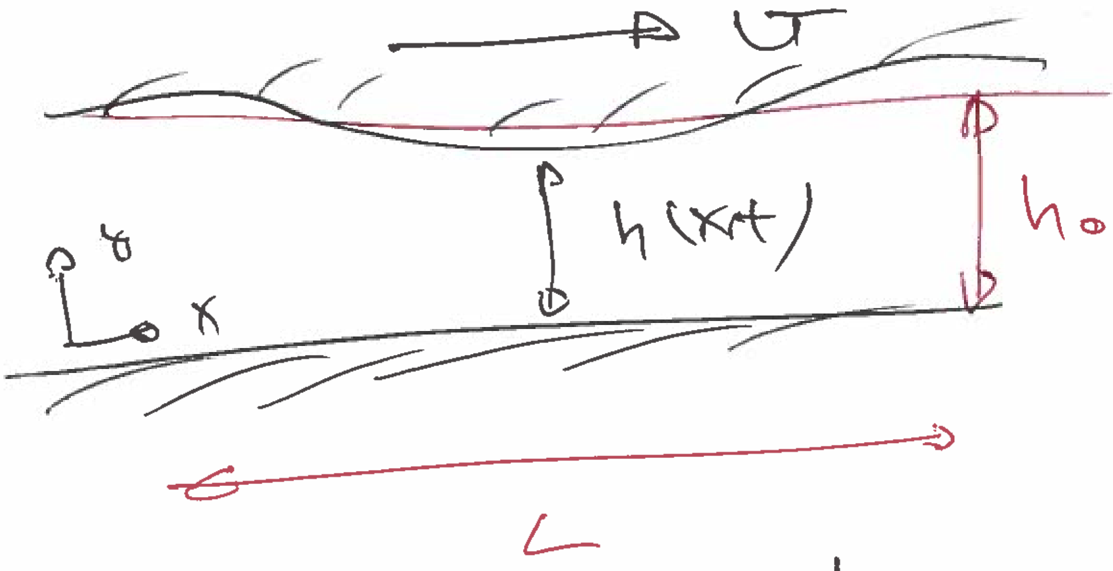


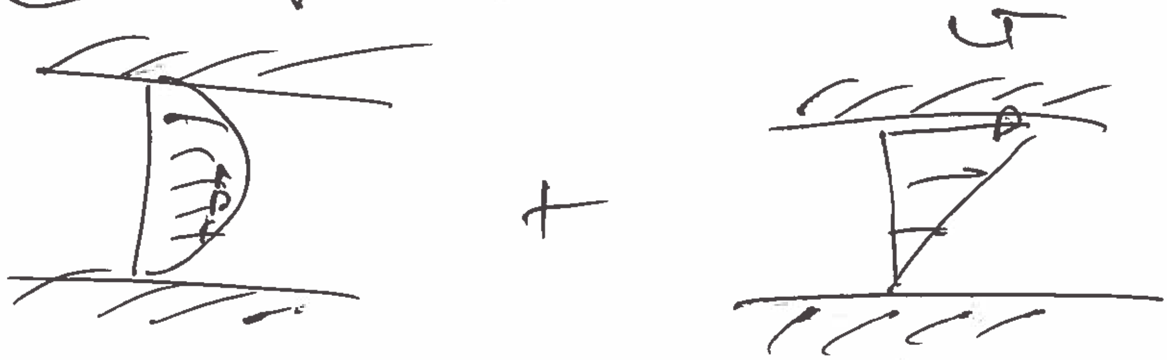
$$\frac{h_0}{L} \ll 1$$

To leading order in h_0/L:

$$0 = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2}$$

$$0 = \frac{\partial p}{\partial y} \qquad + \text{BC}$$

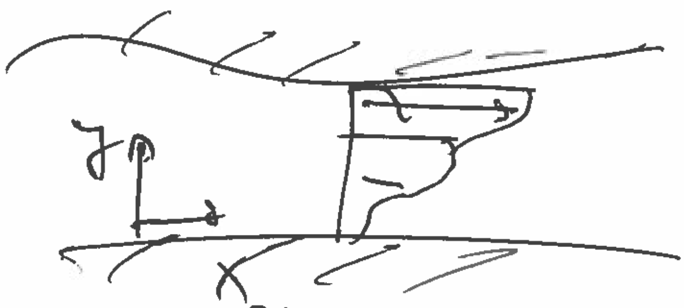
$$\boxed{u(x,y,t) = \underbrace{\frac{1}{2\mu}\frac{\partial p}{\partial x}\left(y^2 - h(x,t)\,y\right)} + \underbrace{U\frac{y}{h(x,t)}}} \quad (**)$$

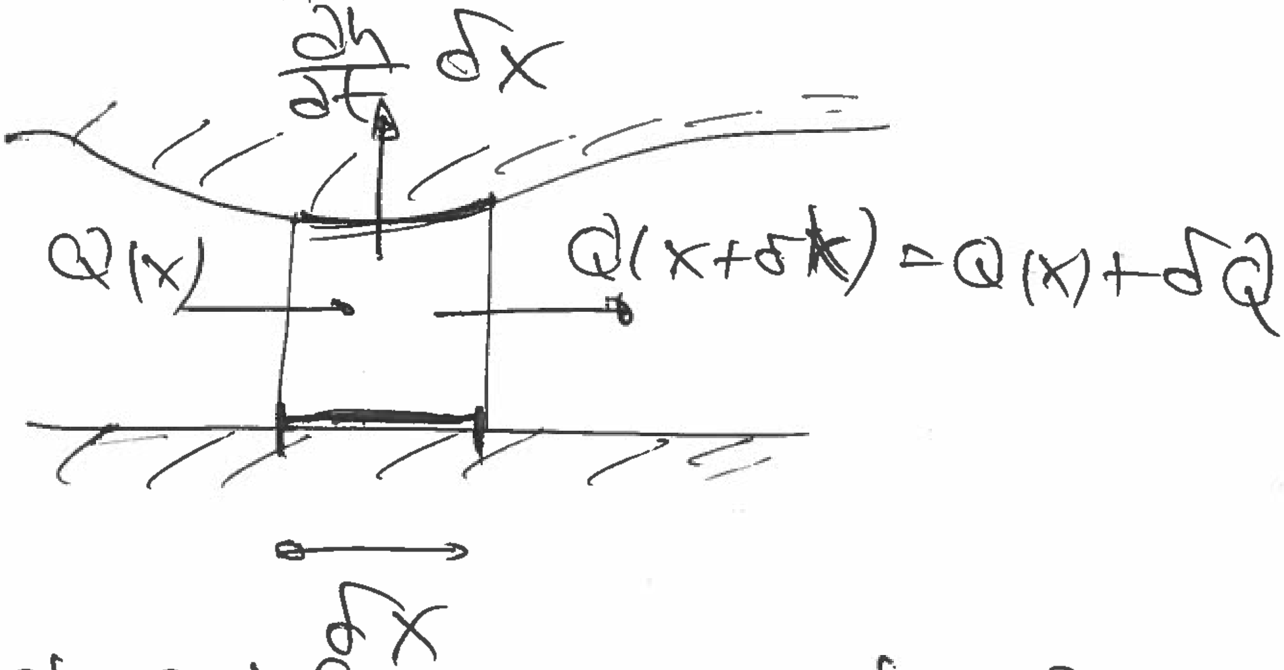


$\frac{\partial p}{\partial x} = ?$ Need conservation of mass!

Consider: Volume flux across the channel

$$Q(x) = \int_0^{h(x,t)} u \, dy$$





Net outflow must be zero:

$$\delta Q + \frac{\partial h}{\partial t}\delta x = 0$$

$$\boxed{\frac{\partial Q}{\partial x} = -\frac{\partial h}{\partial t}} \quad (*)$$

$$Q = \int_0^{h(x,t)} \underbrace{u(x,y,t)}_{\text{see above}} \, dy$$

$$Q(x,t) = -\frac{1}{12\mu}\frac{\partial p}{\partial x}h^3 + \frac{1}{2}Uh$$

into (*)

$$\frac{\partial Q}{\partial x} = \boxed{-\frac{\partial}{\partial x}\left(\frac{1}{12\mu}\frac{\partial p}{\partial x}h^3\right) + \frac{1}{2}U\frac{\partial h}{\partial x} = -\frac{\partial h}{\partial t}}$$

$$\boxed{\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right) = 12\frac{\partial h}{\partial t} + 6U\frac{\partial h}{\partial x}}$$

given

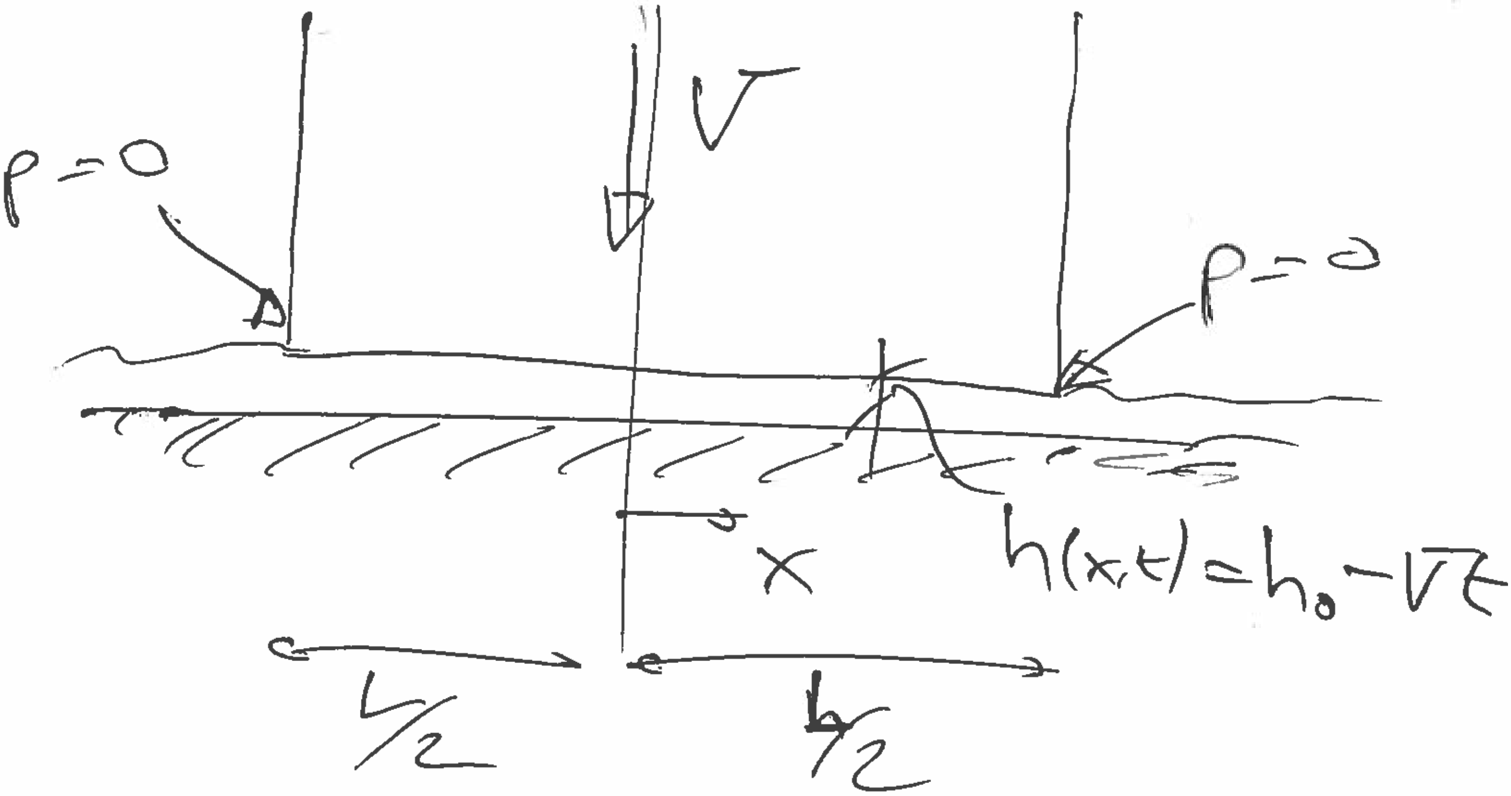
2nd order PDE for $p(x,t)$.

Reynolds Lubrication eqn.

Solve this to get $p(x,t)$ &

then obtain $u(x,y,t)$ from (4)
(**)

Example: Squeeze film



$\frac{\partial h}{\partial x}=0 \; ; \; U=0$

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right)=12\underbrace{\frac{\partial h}{\partial t}}_{-V}$$

$$\frac{h^3}{\mu}\frac{\partial p}{\partial x}=12\frac{\partial h}{\partial t}x+\tilde{A}(t)$$

$$\frac{dp}{dx} = \frac{12\mu\frac{\partial h}{\partial t}}{h^3}x + \underbrace{\frac{\tilde{A}(t)\,\mu}{h^3}}_{A(t)}$$

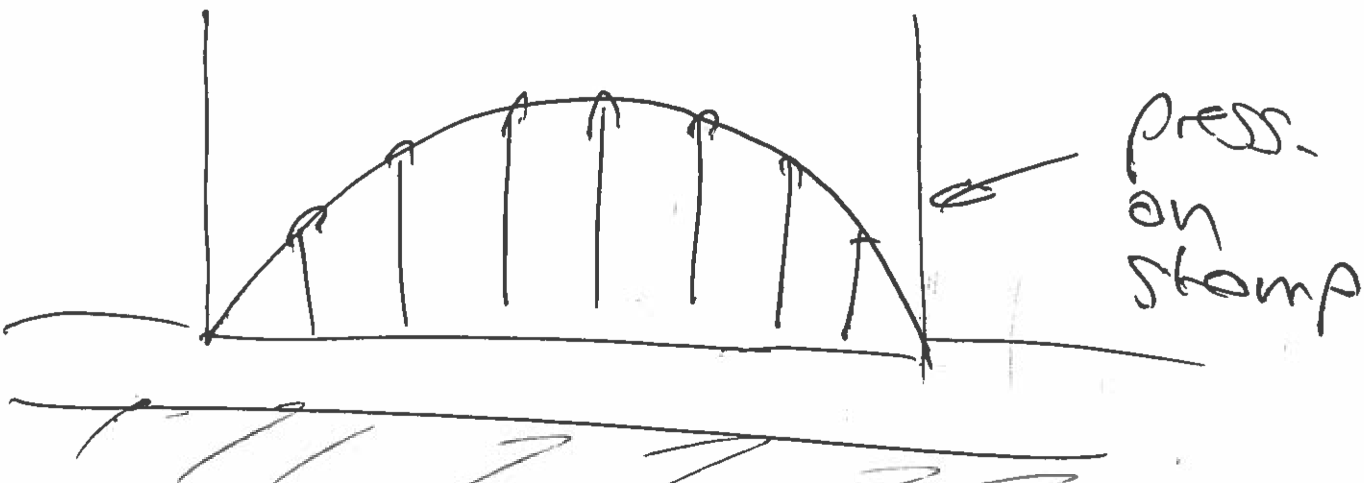
$$p(x,t) = \frac{6\mu\frac{\partial h}{\partial t}}{h^3}x^2 + A(t)x + B(t)$$

BC: $p(x=-\frac{L}{2},t) = p(x=\frac{L}{2},t) = 0$

$$\boxed{p(x,t) = \frac{6\frac{\partial h}{\partial t}\mu}{h^3}\left(x^2 - \left(\frac{L}{2}\right)^2\right)}$$

where: $\frac{\partial h}{\partial t} = -V$

$h = h_0 - Vt$



<u>Note:</u> press. increases rapidly as $h(x,t) \to 0$ (as $t \to \frac{h_0}{V}$)

$\Rightarrow \infty$ force would be req'd to close the gap.

<u>Alternatively:</u> If the stamp is driven by a constant force then $\frac{\partial h}{\partial t} \to 0$ as $h \to 0$.

THANK YOU!