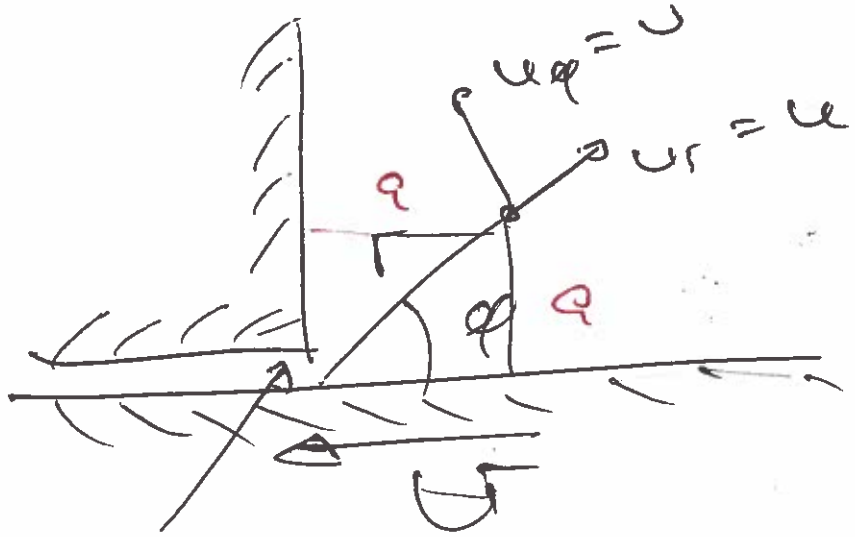


no sep!

$$Re = \frac{\rho U a}{\mu} \ll 1$$

u is indep. of r!

(II) Traction on bottom wall

$$\underline{n} = \overset{0}{n_r}\,\underline{e_r} + \overset{1}{n_\varphi}\,\underline{e_\varphi}$$

$\varphi = 0$

want: tangential traction on wall

$\tau_{r\varphi}$ at $\varphi = 0$

$$f = \left| \int_0^\infty \tau_{r\varphi} \, dr \right|$$

f force per unit depth (in z-direction) on plate.

$$\tau_{ij} = -p\delta_{ij} + 2\mu e_{ij}$$

$$i,j = \{r, \varphi\}$$

$$\tau_{r\varphi} = 2\mu e_{r\varphi}$$

Handout:

$$\frac{\tau_{r\varphi}}{\mu} = r\frac{\partial}{\partial r}\left(\frac{v}{r}\right) + \frac{1}{r}\frac{\partial u}{\partial \varphi}$$

Recall: u & v in dep. of r.

$$r\frac{\partial}{\partial r}\left(\frac{v}{r}\right) \sim r\, r^{-2} \sim \frac{1}{r}$$

$$\frac{1}{r}\frac{\partial u}{\partial \varphi} \sim \frac{1}{r}$$

$$\tau_{r\phi} \sim \frac{1}{r}$$

$$F = \left| \int_0^\infty \tau_{r\phi}\, dr \right| = \infty$$

Force on the infinite plate is infinite!

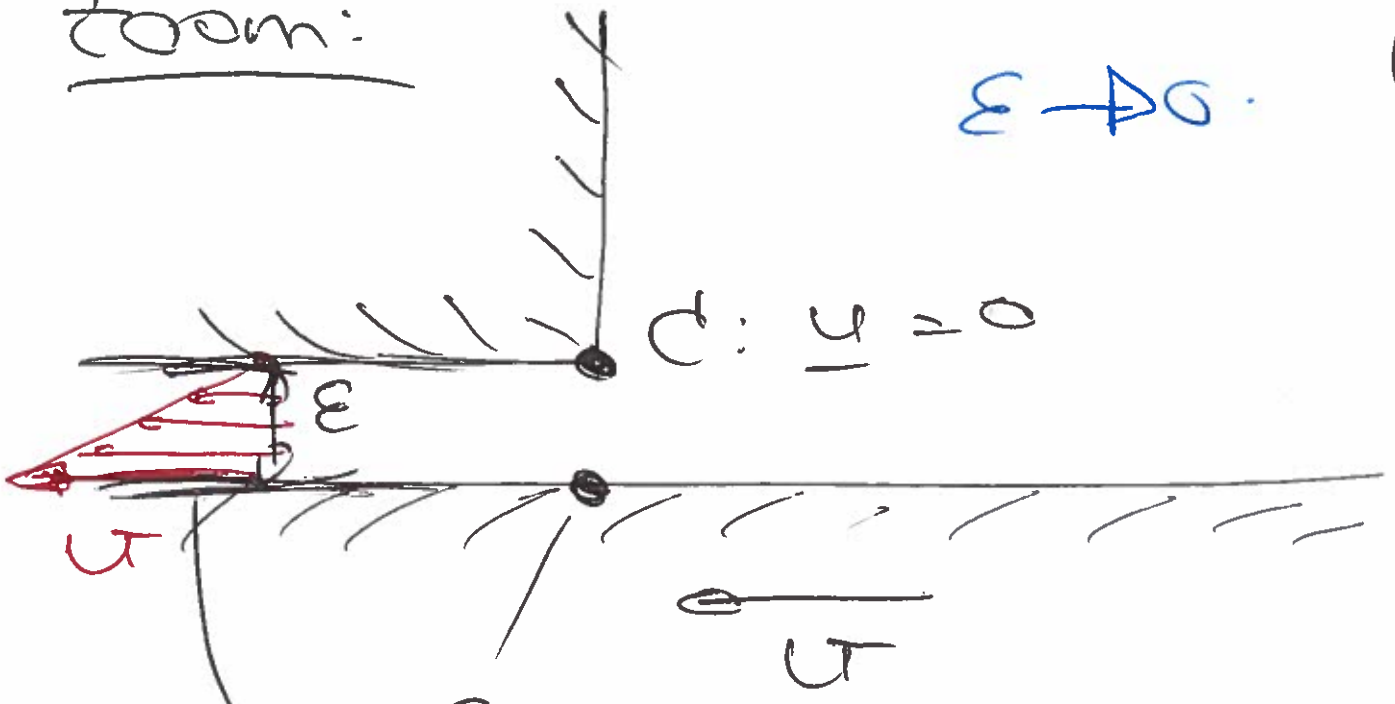
$$F_L = \left| \int_0^L \tau_{r\phi}\, dr \right| = \infty$$

Force on finite part of plate is also ∞.

Hint: Singularity only at $r = 0$.

Zoom:

$\varepsilon \to 0$.



$\varepsilon = 0$:

$$|\tau_{xy}| = \mu \frac{U}{\varepsilon} \to \infty$$

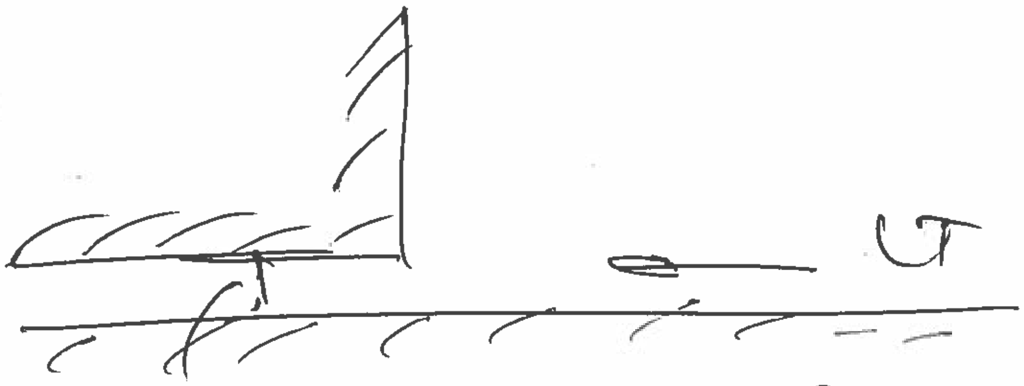
as $\varepsilon \to 0$

Alternatively:

$\varepsilon \to 0$: C → B : veloc @ the corner has to be both 0 & $-U e_x$

⟹ inconsistency resolved by a singularity.

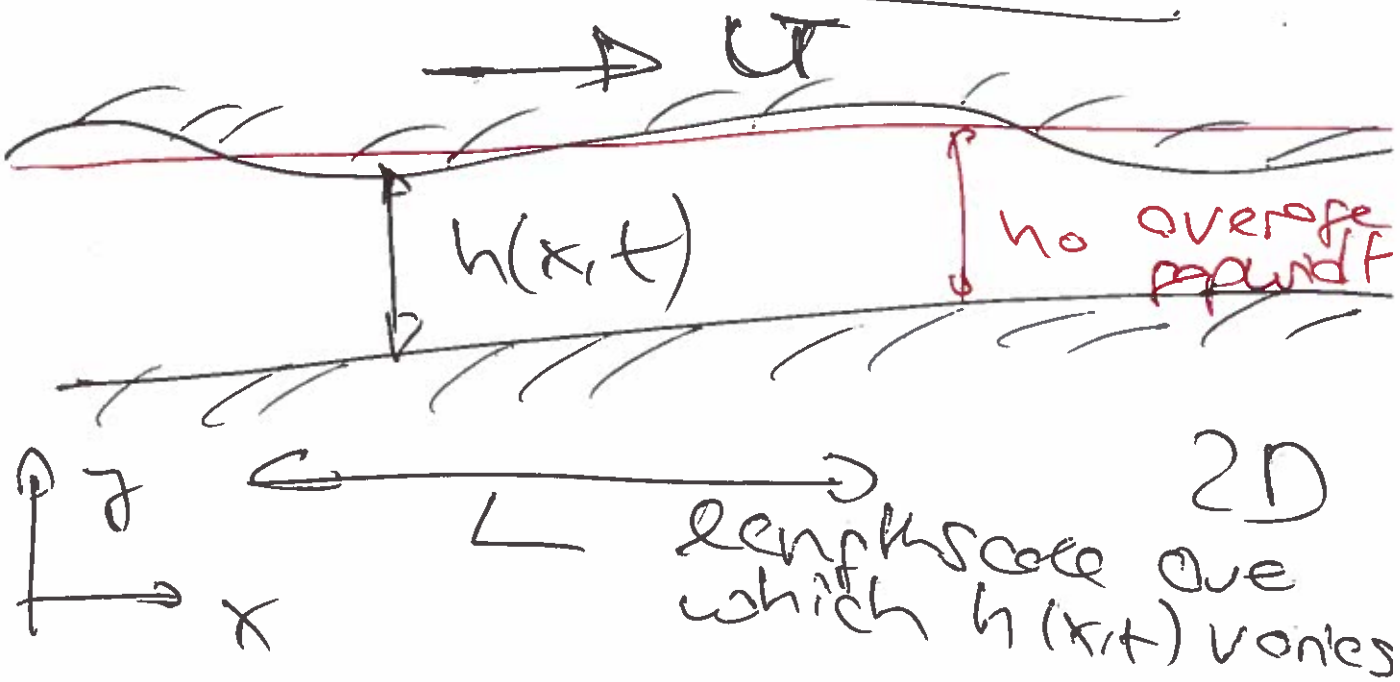
In reality: small gap. 5



ε small but finite

Find chapter!

Lubrication theory



Assume: gap narrow,

$h_0 \ll L$,

and gently varying

$$\left|\frac{\partial h}{\partial x}\right| \ll 1$$

$$\left|\frac{\partial h}{\partial x}\right| = O\left(\frac{h_0}{L}\right) \ll 1$$