Fluid mechanics

L

3 steps:

I Describe mathematically the flow field / motion of fluid particles : Kinematics.

II Formulate the equation of motion (balance of forces acting on fluid particles) : Stresses.

III Constitutive eqns. relate kinematics to stresses.

} Navier Stokes eqns !

+ examples.

Kinematics

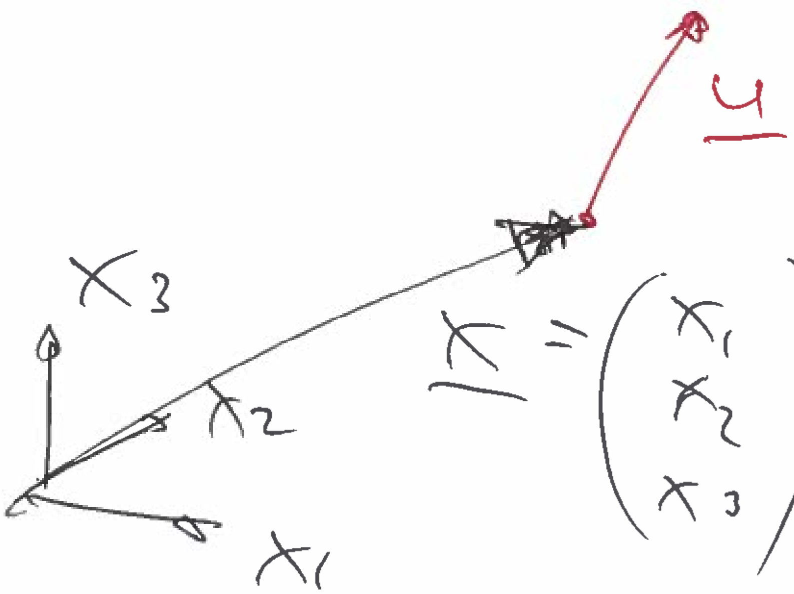
The Eulerian flow field

Assume we know the velocity $\underline{u}$ as a fct. of the three cartesian coordinates (x_1, x_2, x_3) & time t.

$$\underline{u}(x_1, x_2, x_3, t) = \underline{u}(\underline{x}, t)$$

$$u_i(x_j, t)$$

@ sloppy notation.



$$\underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

At time t the material fluid particle at this point has velocity $\underline{u}$.

<u>Note:</u> At different times different fluid particles will be at this position. (Eulerian viewpoint).

This has important implications.

<u>E.g:</u> Acceleration of fluid particles

<u>The material derivative</u>

Assume we are given the position $\underline{x}$ of a fixed ~~point~~ particle:

$$\underline{x} = \underline{x}^p(t) = \begin{pmatrix} x_1^p(t) \\ x_2^p(t) \\ x_3^p(t) \end{pmatrix}$$

The instantaneous velocity of this particle is:

$$\underline{u}(\underline{x}^p(t), t)$$

$$\underline{u}(x_1^p(t), x_2^p(t), x_3^p(t), t)$$

The accel. of the particle is

$$\frac{d\underline{u}}{dt} = \frac{\partial \underline{u}}{\partial t} + \left.\frac{\partial \underline{u}}{\partial x_1}\right|_p \frac{d x_1^p}{dt} + \left.\frac{\partial \underline{u}}{\partial x_2}\right|_p \frac{d x_2^p}{dt} + \left.\frac{\partial \underline{u}}{\partial x_3}\right|_p \frac{d x_3^p}{dt}$$

$$\frac{du_i}{dt} = \frac{\partial u_i}{\partial t} + \frac{\partial u_i}{\partial x_j} \frac{d x_j^p}{dt}$$

u_j

$$\frac{du_i}{dt} = \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j}$$

$$\frac{d\underline{u}}{dt} = \frac{\partial \underline{u}}{\partial t} + \underline{u} \cdot \nabla \underline{u}$$

often written as

$$\frac{D\underline{u}}{Dt}$$ to indicate the material derivative.

The role of strain tensor & the vorticity.

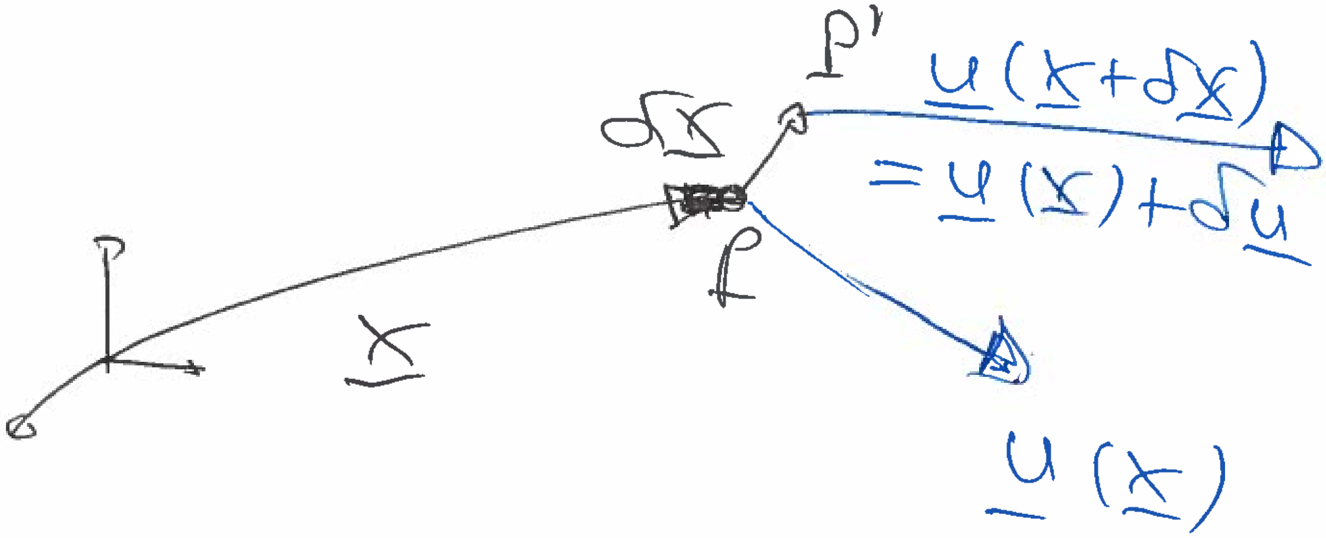
Velocity field itself is not very interesting in itself.
It contains:

- Translation
- Rotation
- Shearing
- Dilation

} motions

How do we identify these?

Examine the veloc. field in the vicinity of a point P.



Taylor expand

$$\underline{u}(\underline{x}+\delta x) = \underline{u}(x_1+\delta x_1,\ x_2+\delta x_2,\ x_3+\delta x_3)$$

$$= \underline{u}(x_1,x_2,x_3) + \underbrace{\frac{\partial \underline{u}}{\partial x_1}\delta x_1 + \frac{\partial \underline{u}}{\partial x_2}\delta x_2 + \frac{\partial \underline{u}}{\partial x_3}\delta x_3}_{\delta \underline{u}} + \ldots$$

$$\delta u_i = \frac{\partial u_i}{\partial x_j}\delta x_j + \ldots$$

Now let $\delta \underline{x} \to \underline{0}$

$$du_i = \underbrace{\frac{\partial u_i}{\partial x_j}}_{\text{velocity gradient tensor}} dx_j$$

velocity gradient tensor 3x3 matrix.

If $\frac{\partial u_i}{\partial x_j} = 0$ then all particles have the same veloc. $\Rightarrow$ pure translation.

The other "modes" of motion must be contained in $\frac{\partial u_i}{\partial x_j}$.