

$$\underline{u} = u(r) \underline{e}_\varphi$$

$$t_i = \tau_{ij} n_j ; \tau_{ij} = -p \delta_{ij} + 2\mu e_{ij}$$

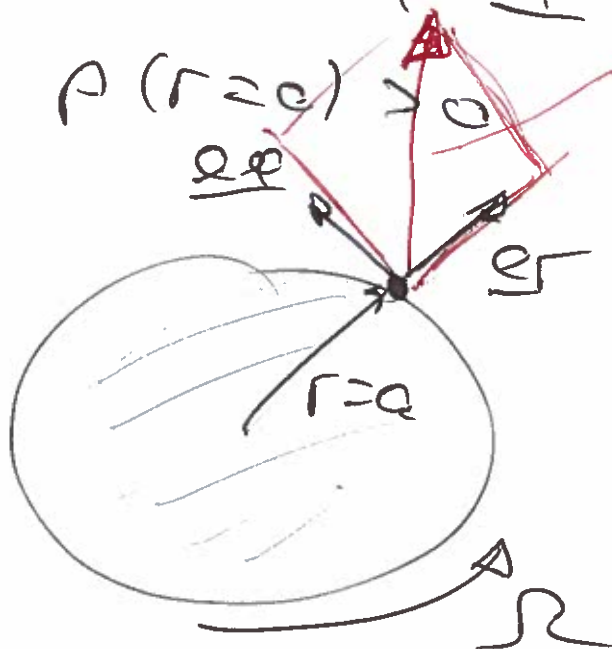
$$\{i, j\} = \{r, \varphi\}$$

$$\left. \begin{aligned} t_r &= p(r=a) \\ t_\varphi &= 2\mu \frac{b^2 \Omega}{b^2 - a^2} \end{aligned} \right\}$$

Traction
on fluid
at $r=a$.

$$\underline{t} = t_r \underline{e}_r + t_\varphi \underline{e}_\varphi$$

Assume: $p(r=0) > 0$



Shear
stress
acting on
fluid
drops the

fluid ~~is~~ in the positive ϕ (ϕ direction) $t_{\phi} > 0$.

Traction acting on the inner cylinder (exerted by fluid) is equal & opposite.

$$\underline{t}_{\text{on cy.}} = -t_r \underline{e}_r - t_{\phi} \underline{e}_{\phi}$$

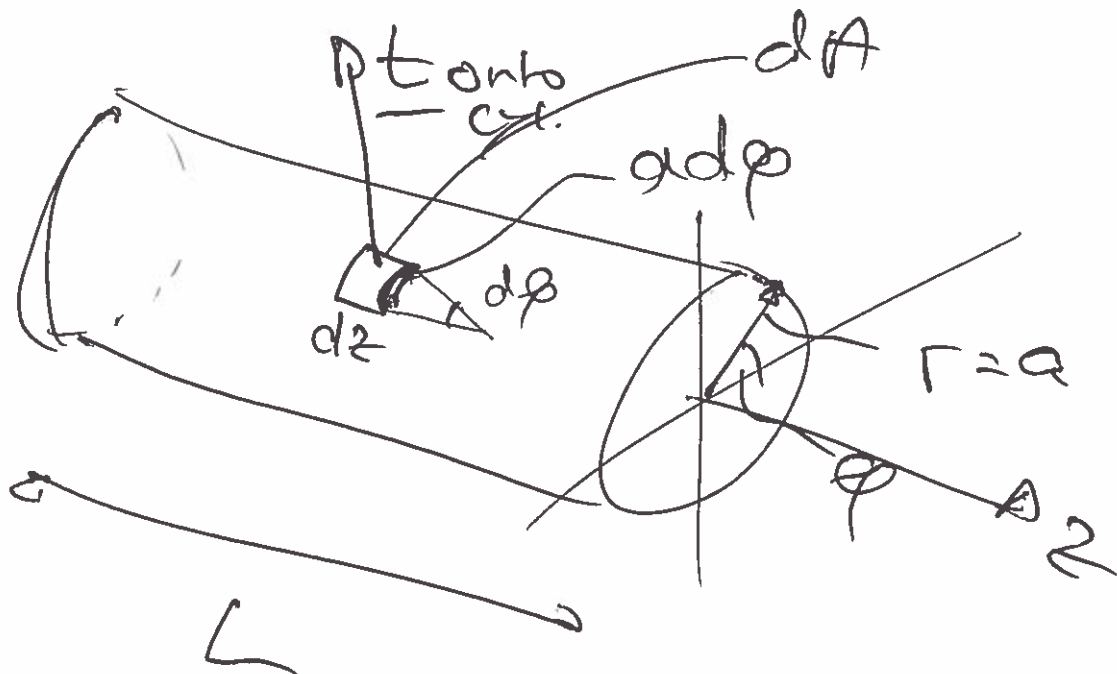
both constant (indep. of ϕ)

force per unit area

\Rightarrow ~~the~~ resultant force acting on a finite length L of the cylinder:

$$\underline{F}_{\text{on cy.}} = \iint_{\text{wetted area}} \underline{t}_{\text{on cy.}} dA$$

~~$$= L \cdot a \int \underline{t}_{\text{on cy.}} d\phi$$~~



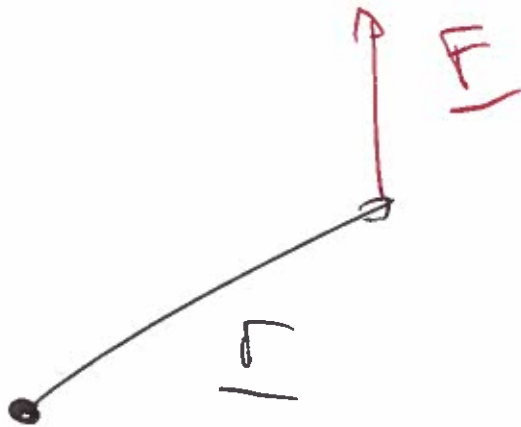
$$dA = a d\phi dz$$

$$\begin{aligned}
 \underline{F}_{\text{onto cyl.}} &= \int_{z=0}^L \int_{\phi=0}^{2\pi} (t_r \underline{e}_r + t_\phi \underline{e}_\phi) a d\phi dz \\
 &= L a \int_{\phi=0}^{2\pi} (t_r \underline{e}_r + t_\phi \underline{e}_\phi) d\phi \\
 &\quad \underbrace{\hspace{10em}}_{\text{const.}}
 \end{aligned}$$

Obvious because of axisymmetry.

Torque on cylinder

(4)

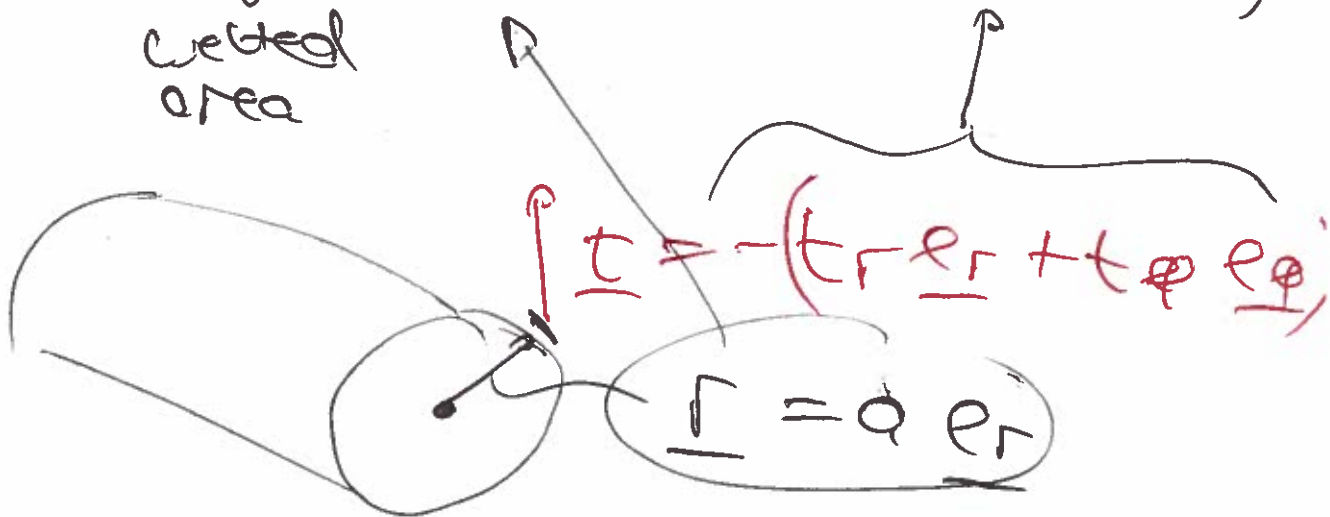


$$\underline{I} = \underline{r} \times \underline{F}$$

for point forces

Generalise to traction

$$\underline{I} = \iint_{\text{cylinder area}} -a \underline{e}_r \times (t_r \underline{e}_r + t_\phi \underline{e}_\phi) dA$$



$$T = \int_{z=0}^{L} \int_{\phi=0}^{2\pi} \underbrace{-a \tau_{\phi} \underline{e}_z}_{\text{indep. of } \phi} a d\phi dz \quad \checkmark$$

$$T = -L a^2 \tau_{\phi} \underline{e}_z$$

$$T = -4\pi \mu L \frac{a^2 b^2}{b^2 - a^2} \underline{e}_z$$

Torque on shaft acts in negative z -direction! True? \checkmark



because the viscosity of the fluid resists the rotation of the inner cylinder.