Example: The vibrating plate: (1

Assume: parallel flow indep. of z, but with time-dependence. Constant pressure. No body force.

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2}$$

(~unsteady heat eqn. ⇒ IVP)

BC: $u(y=0, t) = U\cos(\omega t)$

$u \to 0$ as $y \to \infty$

Find a time-periodic soln.

②

$$u(y,t) = f(y)\cos(\omega t + \theta(y))$$

Instead write:

$$u(y,t) = f(y)\, e^{i\omega t}$$

complex valued

(Then take real part of result).

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2}$$

$$i\omega f = \nu f'' \qquad (\text{cancel } e^{i\omega t})$$

$$f'' - \frac{i\omega}{\nu} f = 0$$

const. coeffn. ODE

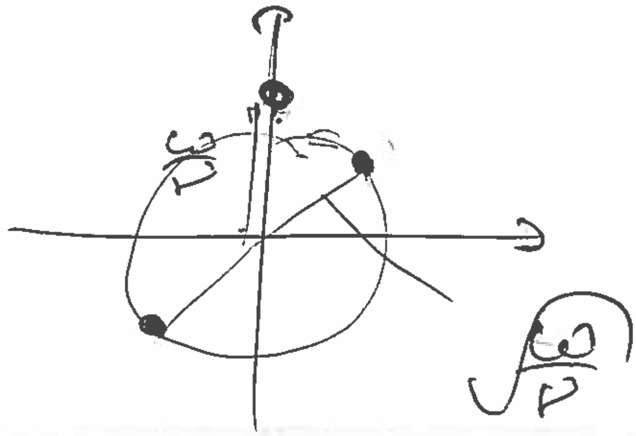
$$f(y) \sim e^{\lambda y}$$

char. poly:

$$\lambda^2 - \frac{i\omega}{\nu} = 0$$

$\lambda = \pm\sqrt{\frac{i\omega}{\nu}}$

$\lambda^2 = \frac{i\omega}{\nu}$



$\lambda_{12} = \pm(1+i)\sqrt{\frac{\omega}{2\nu}}$

$$\boxed{f(y) = A\, e^{(1+i)\sqrt{\frac{\omega}{2\nu}}\, y} + B\, e^{-(1+i)\sqrt{\frac{\omega}{2\nu}}\, y}}$$

arb. constants

BC:

$u(y=0) = U\cos(\omega t)$

$= \mathrm{Re}(U e^{i\omega t})$

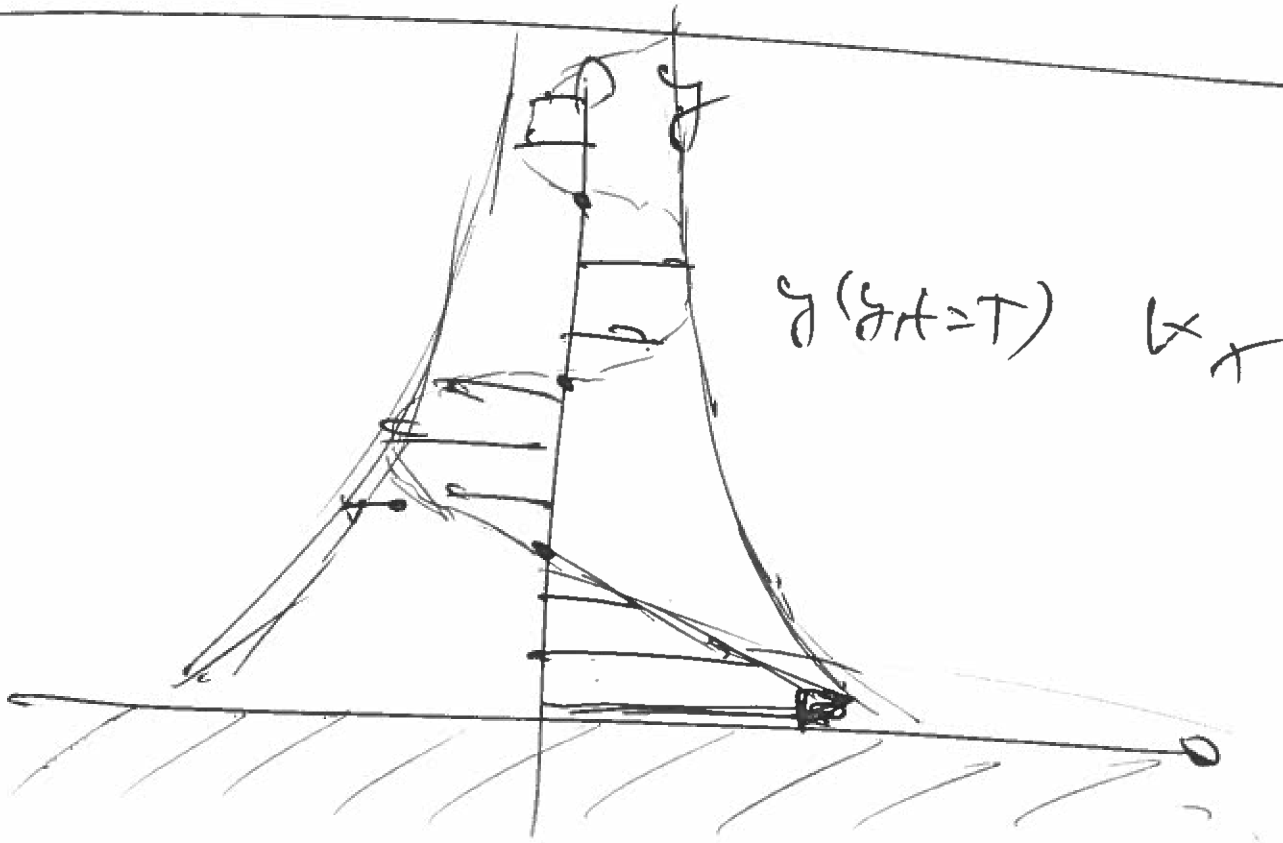
$\Rightarrow \boxed{f(y=0) = U} \boxed{= A + B}$

$u \to 0$ as $y \to \infty$

$f \to 0$ as $y \to \infty \Rightarrow A = 0$

$$u(y,t) = \mathrm{Re}\left(\underbrace{U e^{-(1+i)\sqrt{\frac{\omega}{2\nu}}\,y}}_{f(y)}\, e^{i\omega t}\right)$$

$$\boxed{u(y,t) = U e^{-\sqrt{\frac{\omega}{2\nu}}\,y} \cos\left(\omega t - \sqrt{\frac{\omega}{2\nu}}\,y\right)}$$



See webpage for animation & interpretation.