MATH35001: EXAMPLE SHEET¹ IV

- 1.) The interface between two media is positioned along the plane $x_3 = 0$. Medium (1) is a Newtonian incompressible fluid which occupies the region $x_3 \leq 0$.
 - (i) Assuming that the velocity and pressure fields in the fluid are given, determine the traction that medium (2) exerts onto the fluid along the interface.
 - (ii) Now assume that only the velocity field in the fluid is known and that medium (2) is an inviscid fluid under pressure $p^{(2)} = P$. What is the fluid pressure at the interface? [Remark: The (presumably) unexpected pressure difference is caused by the so-called 'viscous normal stress'].
 - (iii) As in part (i), assume that the velocity and pressure fields in the fluid are known. However, we now replace the inviscid fluid in region (2) by an impermeable stationary solid body. What is the traction exerted onto the fluid at the interface in this case? [Hint: The fluid fulfills the no-slip and no-penetration conditions and the continuity equation along the *entire* interface.] Illustrate the situation with a 2D sketch which shows the direction of the applied traction and the corresponding velocity field near the interface.

Coursework

Please exchange your solution to question 1 with your "marking buddy" and assess each other's work, using the master solution made available on the course webpage (probably in week 5).

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