# MT35001: SOLUTION FOR EXAMPLE SHEET[1] I

**1.)** Which one of these equations in index notation are valid? Remember the summation convention!

- **a)** $c = a_i b_i$ (OK, this is the dot product $c = \mathbf{a} \cdot \mathbf{b}$)
- **b)** $c = a_{ij} b_i$ (Wrong, the free index $j$ doesn't appear on LHS)
- **c)** $c_i = a_{ij} b_i$ (Wrong, the indices on LHS and RHS don't match)
- **d)** $c_i = a_{ij} b_j$ (OK, this is the matrix vector product with the matrix $\underline{\mathbf{a}}$: $\mathbf{c} = \underline{\mathbf{a}}\mathbf{b}$)
- **e)** $c_i = a_{ji} b_j$ (OK, this is the matrix vector product with the transposed matrix $\underline{\mathbf{a}}$: $\mathbf{c} = \underline{\mathbf{a}}^T \mathbf{b}$)
- **f)** $\sigma_{ij} = \alpha_{ij} T + E_{ijkl} e_{kl}$ (Correct – meet your first 4th order tensor. By the way: this is the constitutive equation for a linearly elastic solid incl. temperature variations)
- **g)** $\sigma_{ij} = \alpha_{kl} T_i + E_{ijkl} e_{ij}$ (Wrong, the indices of all terms are different)
- **h)** $k_{ijkl} = a_i b_{kl} c_{njm} d_{mn} + e_{ik} e_{jn} f_{nl}$ (Messy, but correct)

**2.)** Using a comma to denote partial differentiation (e.g. $\partial u/\partial x_2 = u_{,2}$), transform the following expressions into index notation:

- **a)** $\nabla u(x_1, x_2, x_3) \to u_{,i}$
- **b)** $\underline{\mathbf{A}} = \nabla \mathbf{u}(x_1, x_2, x_3) \to a_{ij} = u_{i,j}$
- **c)** $\nabla \cdot \mathbf{u}(x_1, x_2, x_3) = f(x_1, x_2, x_3) \to u_{i,i} = f$
- **d)** $\nabla^2 u(x_1, x_2, x_3) = f(x_1, x_2, x_3) \to u_{,ii} = f$
- **e)** $\nabla^2 \mathbf{u}(x_1, x_2, x_3) = \mathbf{f}(x_1, x_2, x_3) \to u_{i,jj} = f_i$

**3.)** **a)** Show that $\frac{\partial x_i}{\partial x_j} = \delta_{ij}$: Cartesian coordinates are independent of each other, so $\frac{\partial x_i}{\partial x_j} = 1$ if $i = j$ and 0 if $i \neq j$.

**b)** Show that $\delta_{ii} = 3$: Using the summation convention this expands as $\delta_{ii} = \delta_{11} + \delta_{22} + \delta_{33}$, which, given the properties of the Kronecker delta, is equal to three.

**c)** For arbitary vectors $u_i$ and $v_i$ show that

$$S_{ij} = u_i v_j + u_j v_i$$

is symmetric (i.e. $S_{ij} = S_{ji}$) and that

$$T_{ij} = u_i v_j - u_j v_i$$

is antisymmetric (i.e. $T_{ij} = -T_{ji}$). Exchange the indices $i$ and $j$ and re-arrange the terms.

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