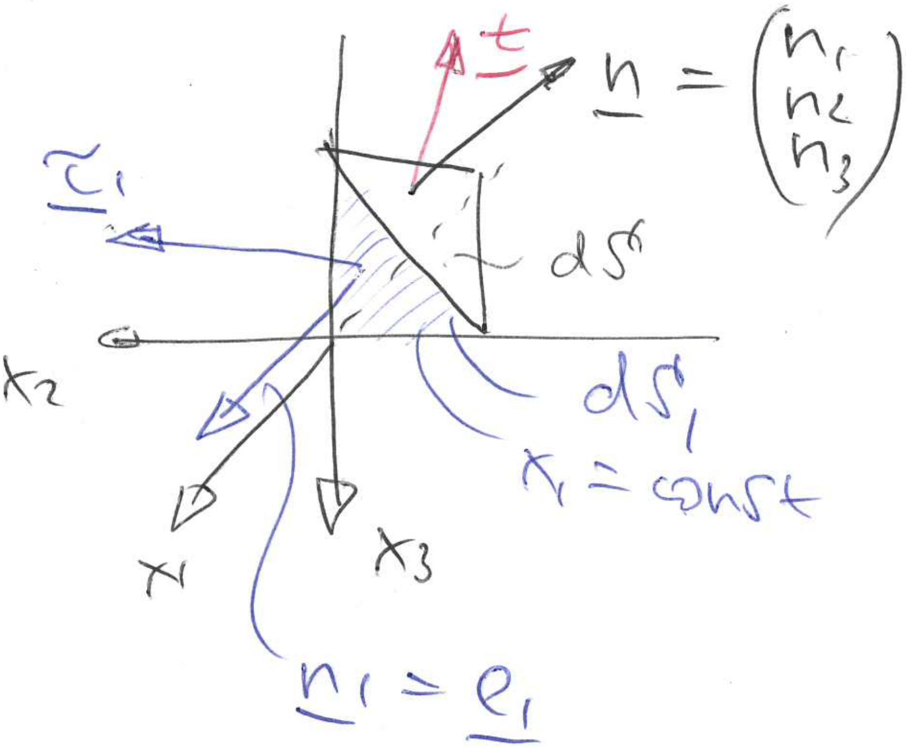


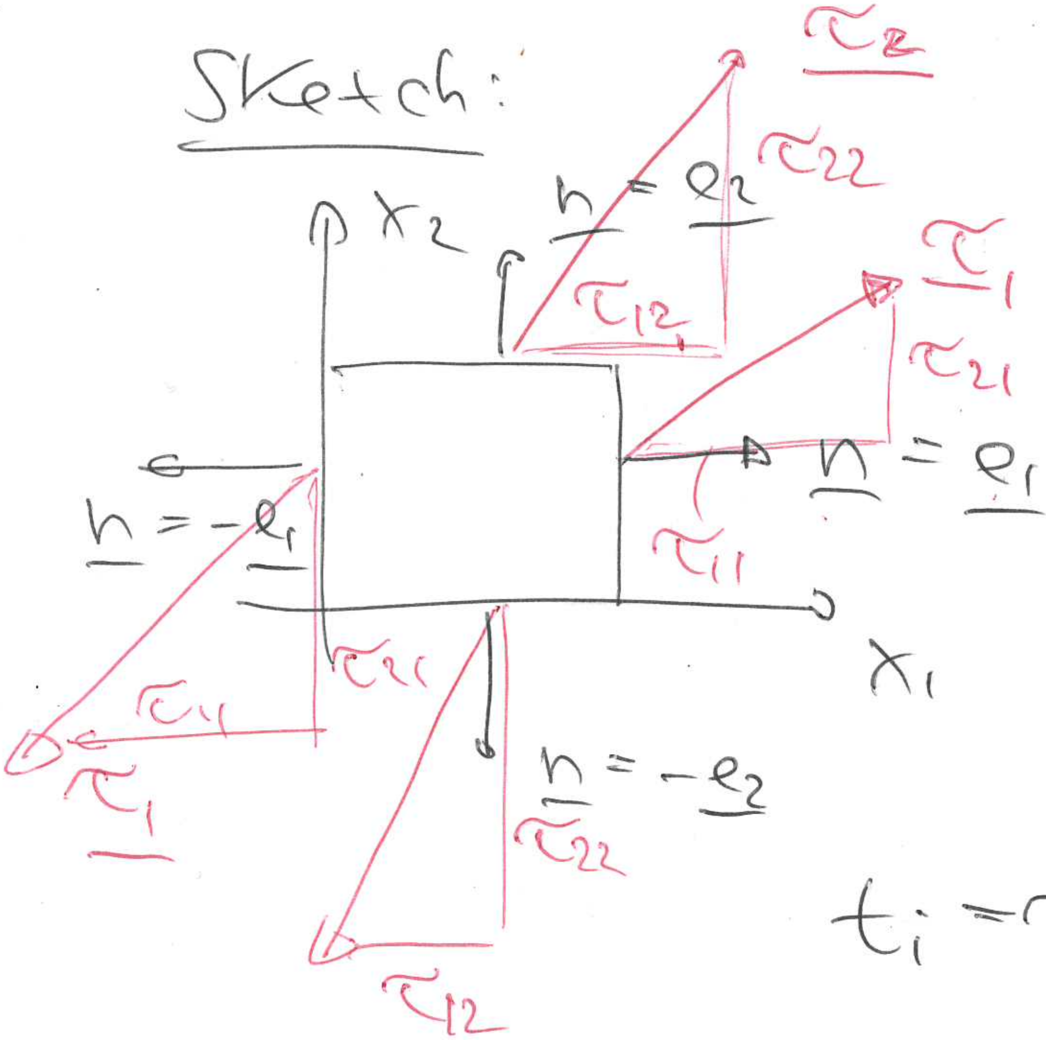
$$\underline{t} = \underline{\tau}_j n_j$$

$$\boxed{t_i = \tau_{ij} n_j}$$

τ_{ij} = stress tensor

τ_{ij} represents the traction/stress component in the x_i-direction acting on the face where x_j = const & whose outer unit normal points in the pos. x_j-direction

Sketch:



$$t_i = \tau_{ij} n_j$$

Can now express stress/traction in terms of $\underline{n}$ & τ_{ij}.

τ_{ij} will soon be expressed as a fct of the rate of strain (constitutive eqn.)

Particular stress states

(i) Hydrostatic pressure

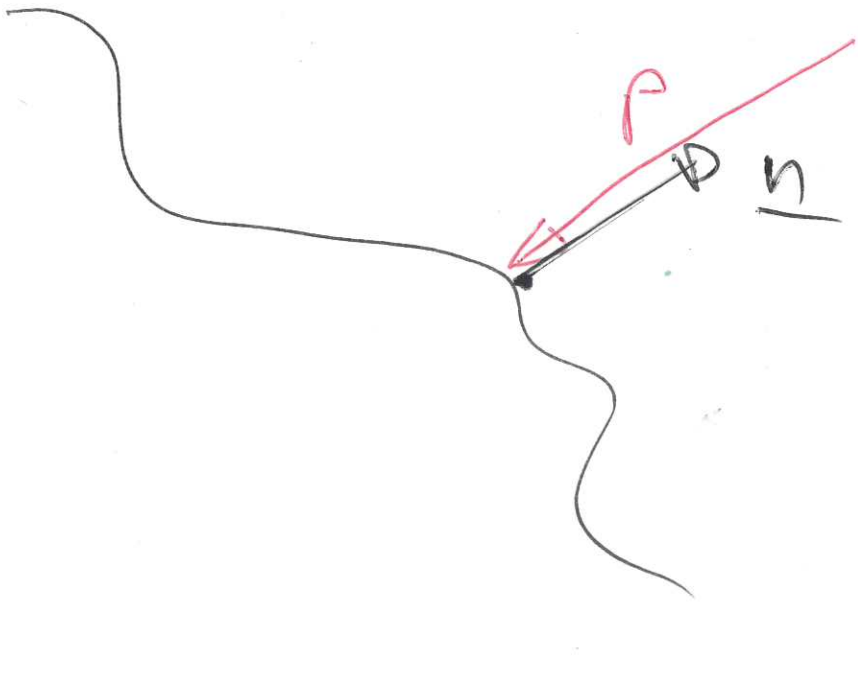
$$\sigma_{ij} = -p\,\delta_{ij}$$

implies that the traction is always normal to $\underline{n}$ & uniform in all directions:

$$t_i = \sigma_{ij} n_j$$

$$t_i = -p\,\underbrace{\delta_{ij} n_j} = -p n_i$$

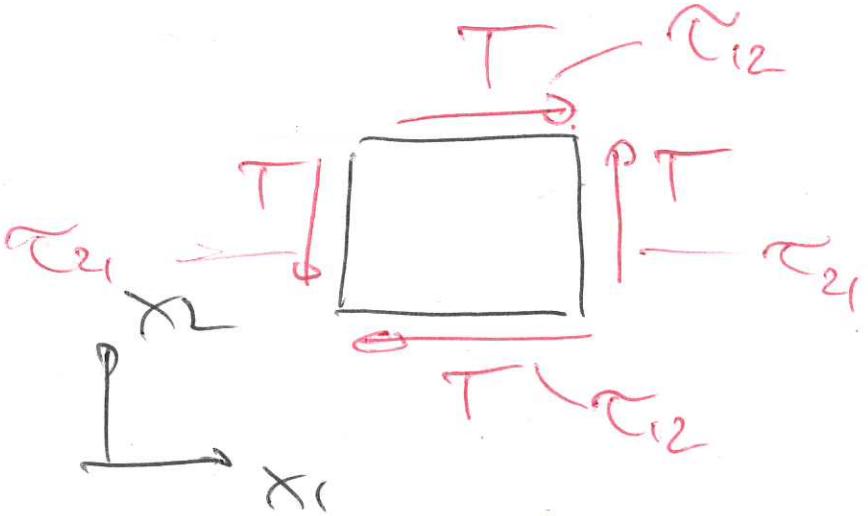
$$\underline{t} = -p\,\underline{n}$$



(ii) pure shear:

e.g. $\tau_{12} = \tau_{21} = T$

$\tau_{ij} = 0$ otherwise

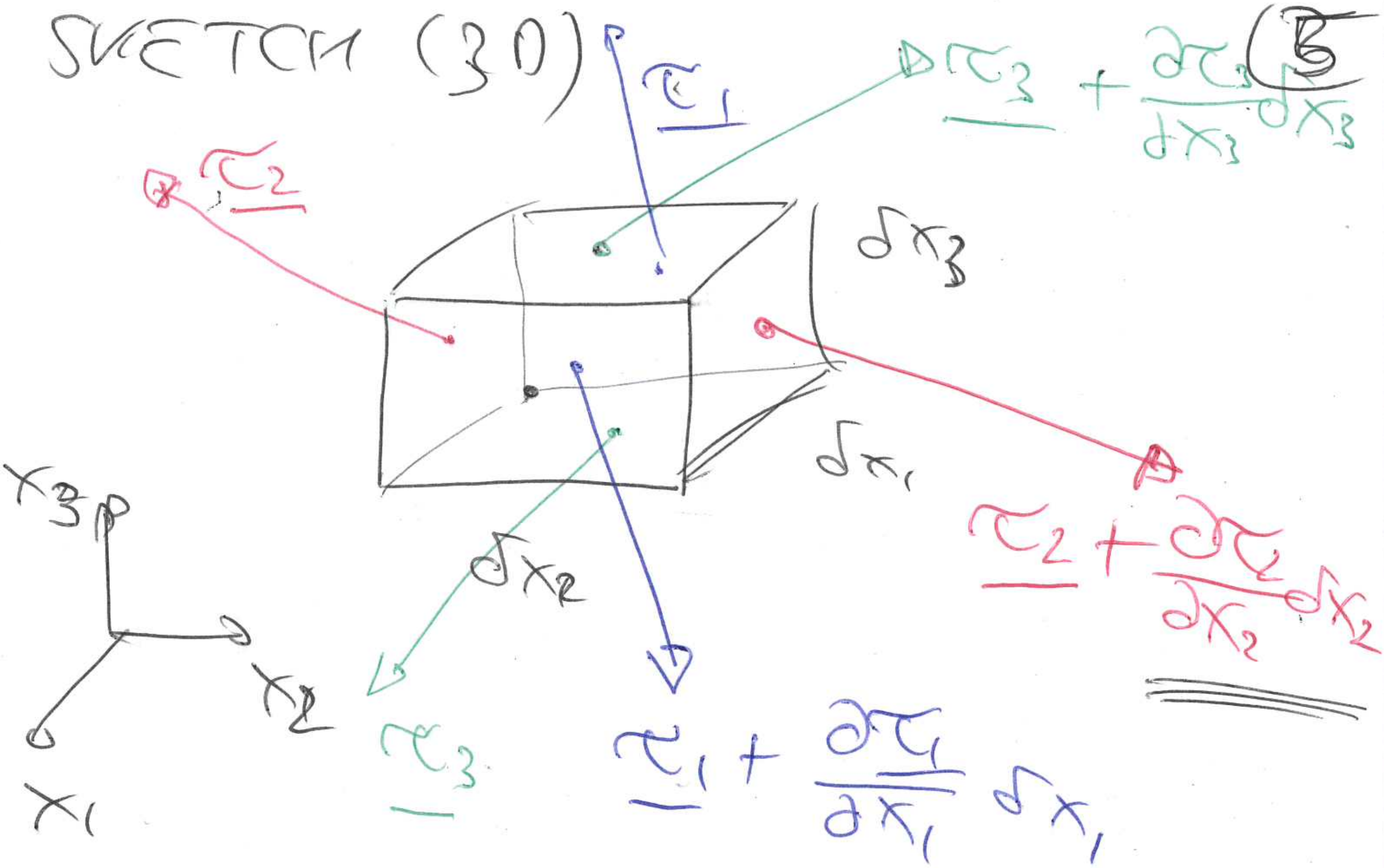


Equilibrium of forces / Cauchy eqn

"$\sum$ forces = mass $\times$ accel"

for an infinitesimal fluid particle.

SKETCH (3D)



only the increment in stress makes a contribution to the sum of all forces!

$$\frac{\partial \underline{\tau}_1}{\partial x_1} \cancel{\delta x_1 \delta x_2 \delta x_3} +$$

$$\frac{\partial \underline{\tau}_2}{\partial x_2} \cancel{\delta x_2 \delta x_1 \delta x_3} +$$

$$\frac{\partial \underline{\tau}_3}{\partial x_3} \cancel{\delta x_3 \delta x_1 \delta x_2} +$$

$$\rho \underline{F} \cancel{\delta x_1 \delta x_2 \delta x_3} =$$

$$= \rho \frac{D u}{D t} \cancel{\delta x_1 \delta x_2 \delta x_3}$$

where $\underline{f}$ is a body force = force per unit mass e.g. gravitational acceleration, $-g\,\underline{e_z}$.

$$\rho \frac{D\underline{u}}{Dt} = \frac{\partial \tau_{ij}}{\partial x_j} + \rho \underline{f}$$

$$\rho \frac{D u_i}{Dt} = \frac{\partial \tau_{ij}}{\partial x_j} + \rho f_i$$

(Cauchy's eqn.)

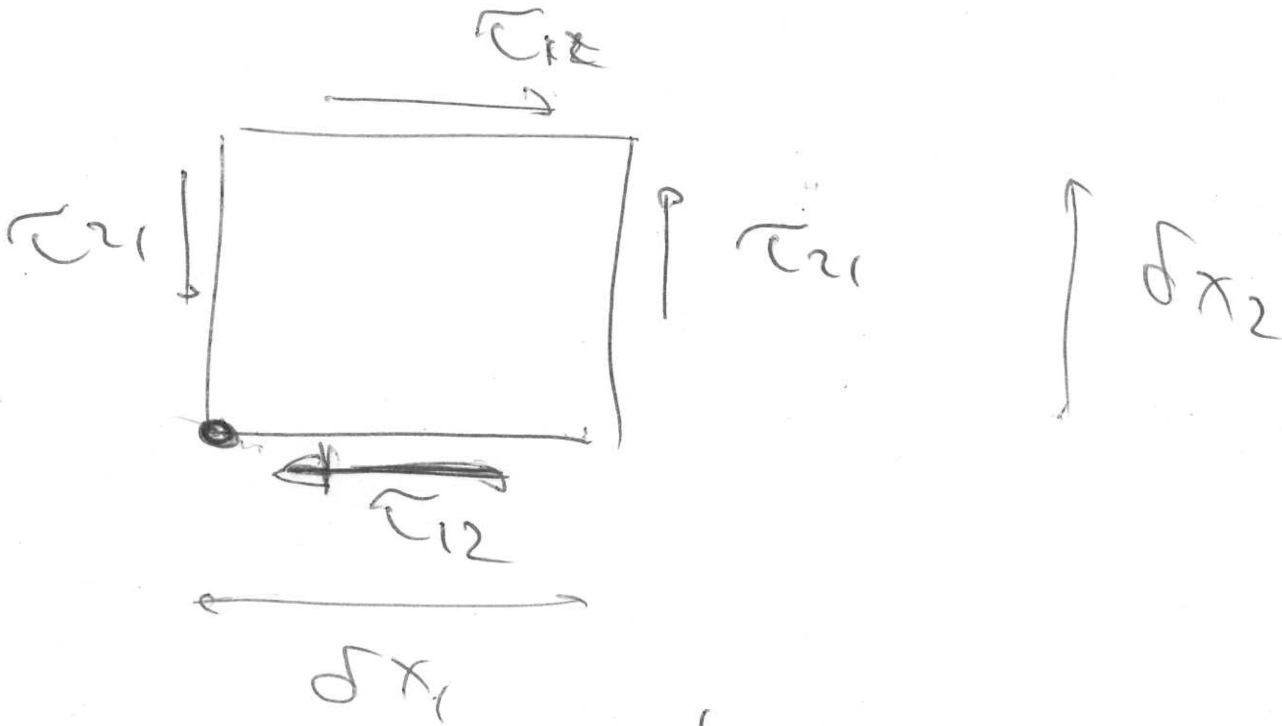
$$\rho \left(\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) = \frac{\partial \tau_{ij}}{\partial x_j} + \rho f_i$$

This encodes rate of change of veloc. as a fct. of the current veloc. & τ_{ij}

Q: How does τ_{ij} depend on $\underline{u}$?

Symmetry of τ_{ij}

$$\tau_{ij} = \tau_{ji}$$



$$\tau_{21} \cancel{\delta x_2} \delta x_1 \overset{!}{=} \tau_{12} \cancel{\delta x_1} \delta x_2$$

$$\tau_{21} = \tau_{12}$$