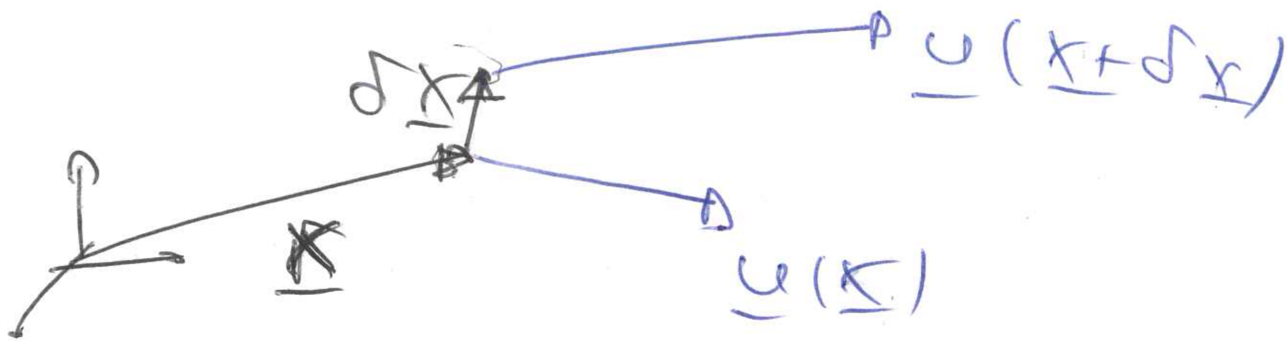


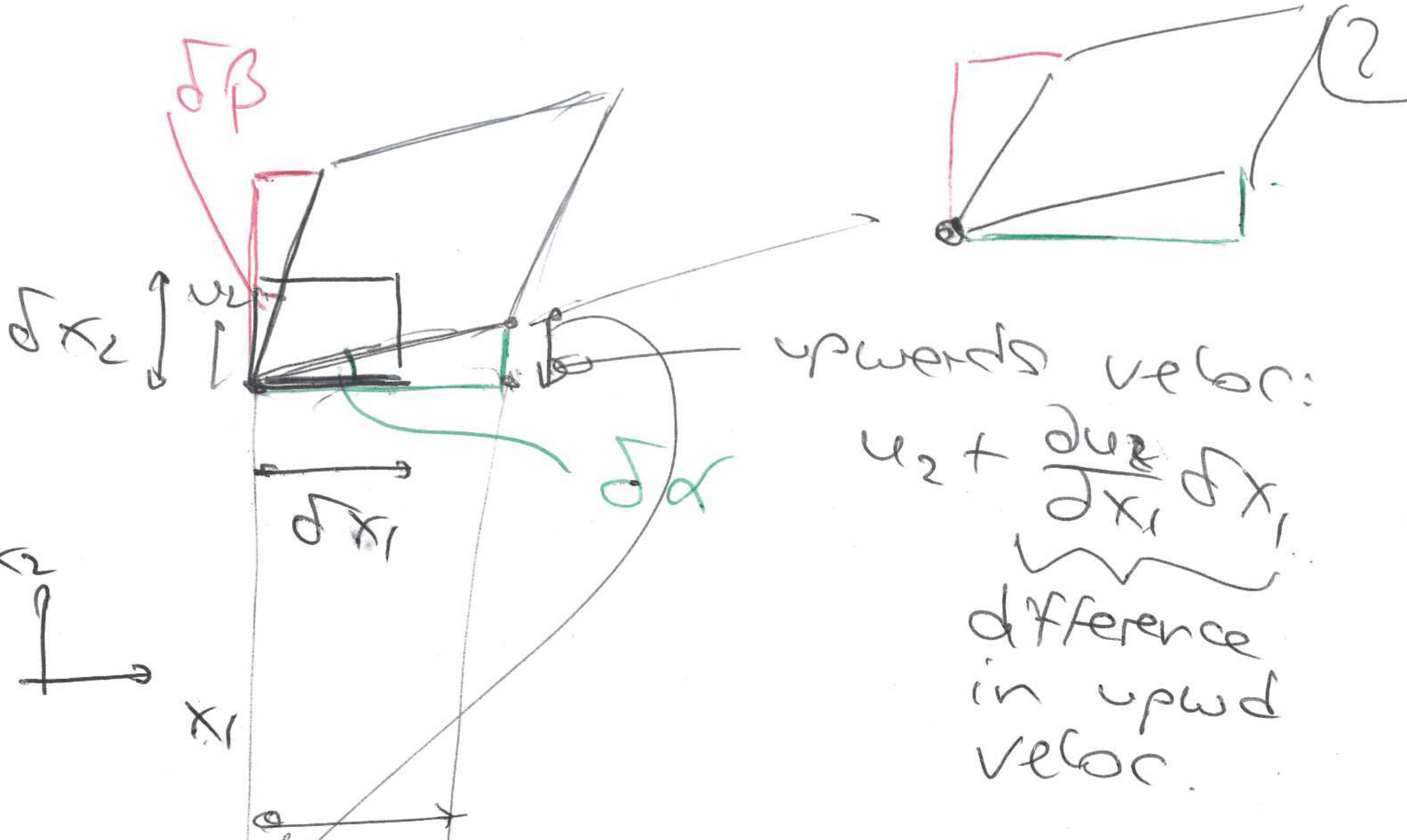
$$\delta\underline{u} = \underline{u}(\underline{x}+\delta\underline{x}) - \underline{u}(\underline{x})$$

$$\delta u_i = \underbrace{\frac{\partial u_i}{\partial x_j}\,\delta x_j}$$

$$= \varepsilon_{ij}\,\delta x_j + \underbrace{\omega_{ij}\,\delta x_j}_{\text{rotation}}$$

$\varepsilon_{(ii)}$ = rete of strain in x_i direction

② Sheer rete of strain



$$\left(1+\frac{\partial u_1}{\partial x_1}\delta t\right)\delta x_1$$

see last lecture

$$\frac{\partial u_2}{\partial x_1}\delta x_1\, \delta t$$

$$\tan \delta\alpha = \frac{\frac{\partial u_2}{\partial x_1}\cancel{\delta x_1}\, \delta t}{\left(1+\cancel{\frac{\partial u_1}{\partial x_1}\delta t}\right)\cancel{\delta x_1}}$$

In the limit of $\delta t \to 0$

$\delta\alpha \to 0$

$\tan(\delta\alpha) \simeq \sin(\delta\alpha) \simeq \delta\alpha$

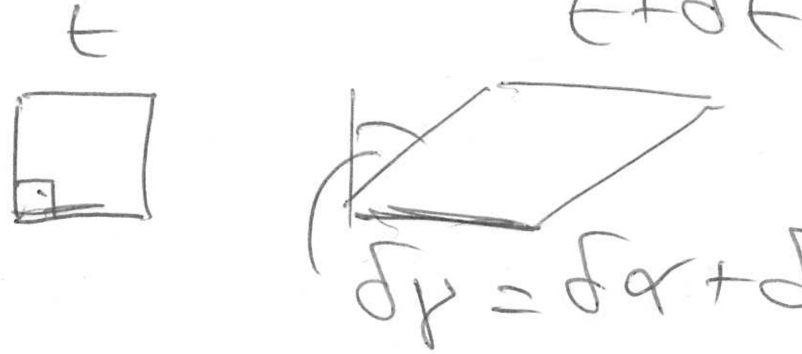
$$\delta\alpha = \frac{\partial u_2}{\partial x_1}\delta t$$

$$\frac{\delta\alpha}{\delta t} = \frac{\partial u_2}{\partial x_1} = \frac{D\alpha}{Dt}$$

Similarly:

$$\frac{D\beta}{Dt} = \frac{\partial u_1}{\partial x_2} \quad \text{exercice}$$

Now consider the "shear rate" i.e. the rate at which the angle between two material lines that were initially aligned with the x_1 & x_2-axes changes.



$$\delta\gamma = \delta\alpha + \delta\beta$$

$$\frac{D\gamma}{Dt} = \frac{D\alpha}{Dt} + \frac{D\beta}{Dt} = \left(\frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2}\right)$$

$$\frac{D\gamma}{Dt} = 2\varepsilon_{12}$$

similar for other coord. directions

$\boxed{\text{The off-diagonal entries of } \varepsilon_{ij} \text{ represent } \underline{\text{half}} \text{ the shear rate in the } x_i - x_j \text{ plane}}$

Summary:

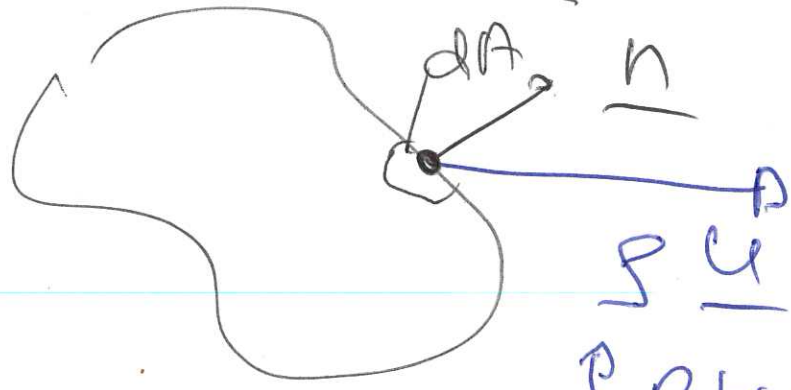
Motion of fluid in the vicinity of a fixed spatial point can be decomposed into:

$$\underline{u}(\underline{x}+\delta\underline{x}) = \underbrace{\underline{u}(\underline{x})}_{\text{translation}} + \underbrace{\underline{\Omega}\times\delta\underline{x}}_{\text{rotation}} + \underbrace{\underset{\sim}{E}\,\delta\underline{x}}_{\text{dilatation/stretching \& shearing}}$$

Eqn. of continuity

Physics: mass flux into a spatially fixed volume = rate of change of mass in that volume.

Integral form



ρ RHO = density (kg/m^3)

Mass flux: [$\frac{kg}{sec}$]
density ρ [$\frac{kg}{m^3}$] × veloc. normal to the surface [$\frac{m}{sec}$] × cross-sectional (surface) area [m^2]

$$\boxed{-\oint \rho \underline{u} \cdot \underline{n} \, dA = \int \frac{d\rho}{dt} dV}$$

Transform to differential form:
Diverfence theorem:

$$\int \frac{d\rho}{dt} dV + \underbrace{\oint (\rho \underline{u}) \cdot \underline{n} \, dA}_{\int \operatorname{div}(\rho \underline{u}) \, dV} = 0$$

$$\int \left(\frac{d\rho}{dt} + \operatorname{div}(\rho \underline{u}) \right) dV = 0$$

If this is true for any volume, the integrand has to vanish everywhere

$$\boxed{\frac{d\rho}{dt} + \operatorname{div}(\rho \underline{u}) = 0}$$