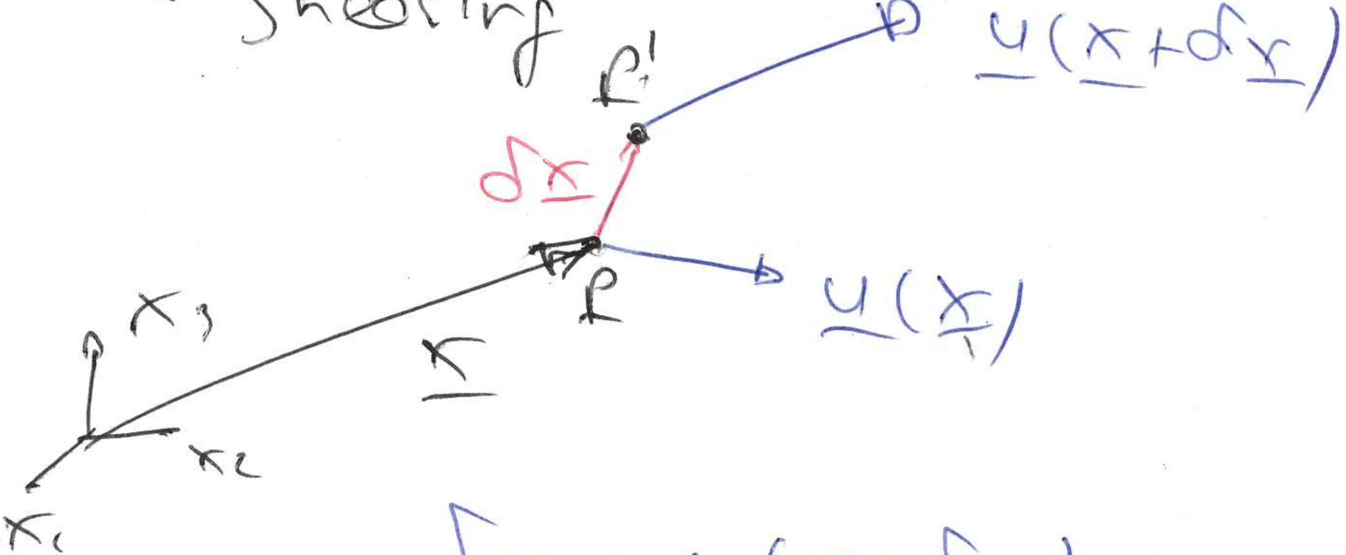
Fluid motion:

$\underline{u}(\underline{x}, t)$

can be decomposed into:

- translation ✓
- rotation
- dilatation
- shearing



$$\delta\underline{u} = \underline{u}(\underline{x}+\delta\underline{x}) - \underline{u}(\underline{x})$$

$$\boxed{\delta u_i = \frac{\partial u_i}{\partial x_j}\,\delta x_j} + \dots$$

$\Rightarrow$ If $\frac{\partial u_i}{\partial x_j} = 0 \Rightarrow$ same veloc. everywhere $\Rightarrow$ <u>translation</u>.

$\frac{\partial u_i}{\partial x_j}$ contains all the "other modes".

To see this split $\frac{\partial u_i}{\partial x_j}$

$$\frac{\partial u_i}{\partial x_j} = \underbrace{\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)}_{\varepsilon_{ij} = \varepsilon_{ji}} + \underbrace{\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right)}_{\omega_{ij} = -\omega_{ji}}$$

ε_{ij}: rate of strain tensor

ω_{ij}: rate of rotation tensor

$$\underbrace{\delta u_i}_{u_i(x_k + \delta x_k) - u_i(x_n)} = \underbrace{\varepsilon_{ij}\,\delta x_j}_{\text{stretching = dilatation \& shearing}} + \underbrace{\omega_{ij}\,\delta x_j}_{\text{rotation}}$$

Rotation / Vorticity

Consider the veloc. increment induced by $\omega_{ij}\,\delta x_j$

$$\omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right) = -\omega_{ji}$$

$$\delta u_i = \omega_{ij}\,\delta x_j$$

$$\begin{pmatrix} \delta u_1 \\ \delta u_2 \\ \delta u_3 \end{pmatrix} = \begin{pmatrix} 0 & -\Omega_3 & \Omega_2 \\ \Omega_3 & 0 & -\Omega_1 \\ -\Omega_2 & \Omega_1 & 0 \end{pmatrix} \begin{pmatrix} \delta x_1 \\ \delta x_2 \\ \delta x_3 \end{pmatrix}$$

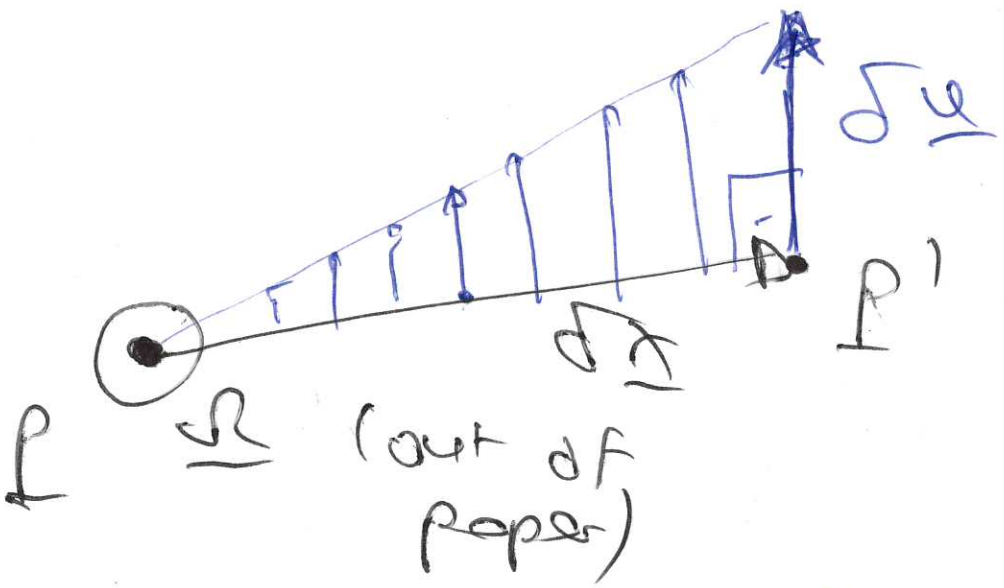
can be written as:

$$\delta \underline{u} = \underline{\Omega} \times \delta \underline{x}$$

where

$$\underline{\Omega} = \begin{pmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{pmatrix}$$ is the rate of rotation vector.

Geometrical interpretation:



q.e.d.

$$\underline{\Omega} = \frac{1}{2} \underbrace{\nabla \times \underline{u}}_{\underline{\omega}} = \frac{1}{2} \begin{pmatrix} \frac{\partial}{\partial x_1} \\ \frac{\partial}{\partial x_2} \\ \frac{\partial}{\partial x_3} \end{pmatrix} \times \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

↳ vorticity.

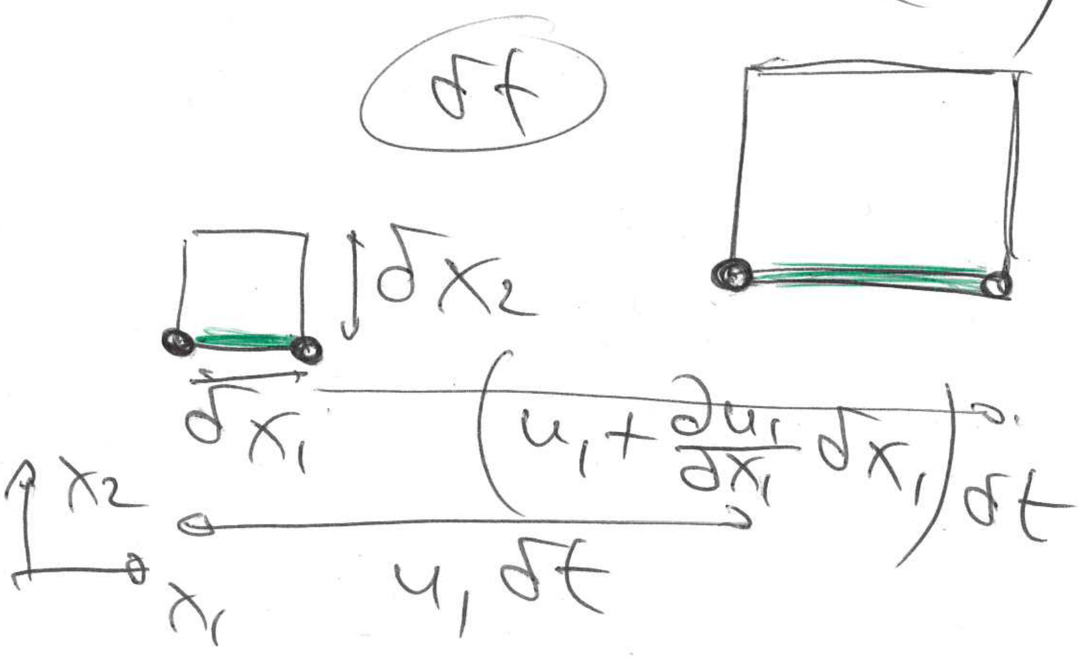
The rate of strain

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

contains shearing & dilatation/stretching.

1 Extensional rate of strain

Illustration (2D)



$$\text{strain} = \frac{\text{length} - \text{orig length}}{\text{orig length}}$$

↳ δx_1

$$\text{strain} = \frac{\overbrace{\cancel{\delta x_1} + \left(\cancel{u_1} + \frac{\partial u_1}{\partial x_1}\cancel{\delta x_1}\right)\delta t - \cancel{u_1 \delta t}}^{\text{new length}} - \cancel{\delta x_1}}{\cancel{\delta x_1}}$$

$$\text{strain} = \frac{\partial u_1}{\partial x_1}\delta t$$

$$\text{rate of strain} = \frac{d\,\text{"strain"}}{dt}$$

$$= \frac{\partial u_1}{\partial x_1} = \varepsilon_{11}$$

Similar for other directions

$\boxed{\text{$\varepsilon_{11}$ etc. (i.e. the diag. entries in the rate of strain tensor) represent the extensional rate of strain in the x_1 etc. direction}}$