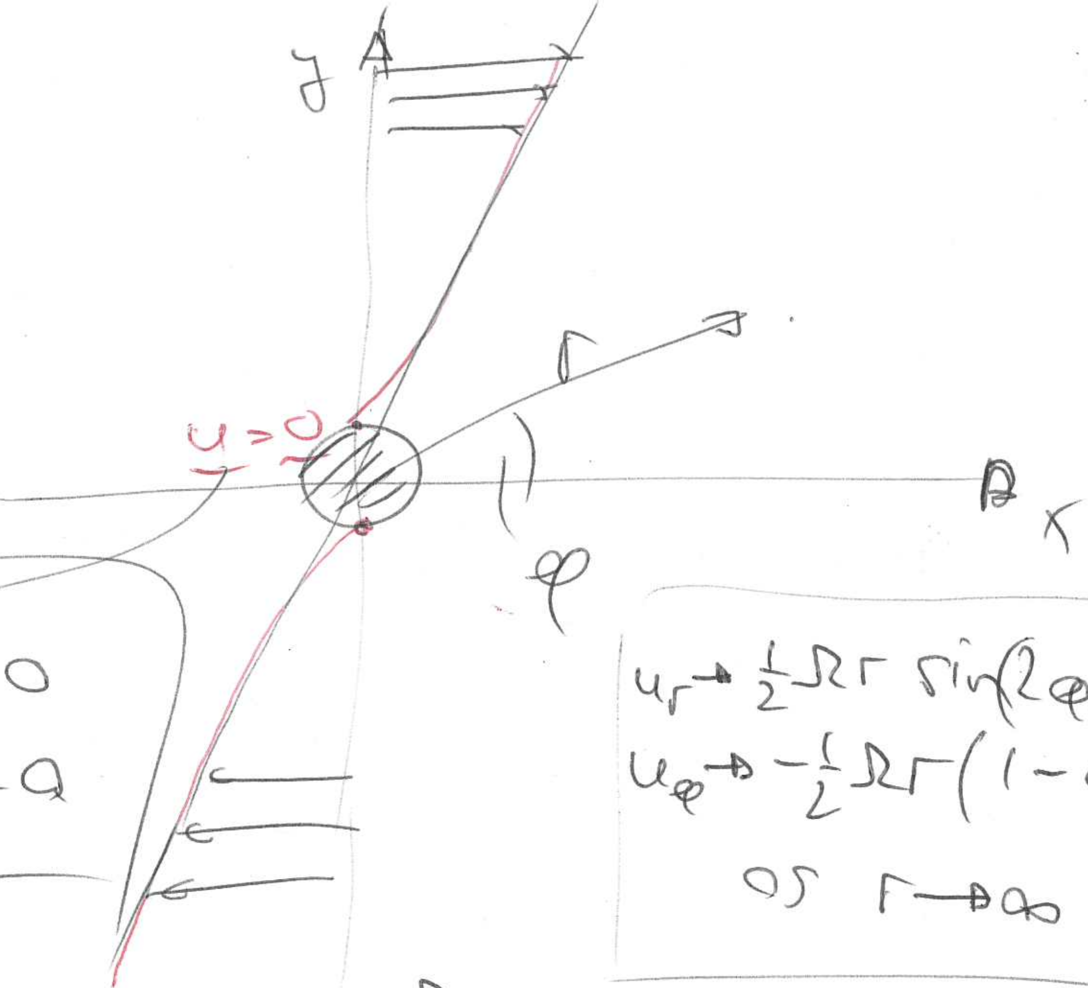


$u_r = u_\varphi = 0$
at $r = a$

$u_r \to \frac{1}{2}\Omega r \sin(2\varphi)$
$u_\varphi \to -\frac{1}{2}\Omega r (1 - \cos(2\varphi))$
as $r \to \infty$

$$\underline{u} \to \Omega y \, \underline{e_x} \quad \text{as } r \to \infty$$

$$\nabla^4 \psi = 0 \, ; \quad \psi(r,\varphi) = F(r) + G(r)\cos(2\varphi)$$

$$\psi(r,\varphi) = \cancel{A_0} + B_0 r^2 + C_0 \ln r + \cancel{D_0 r^2 \ln r} + \left(A_2 r^2 + B_2 \frac{1}{r^2} + \cancel{C_2 r^4} + D_2\right)\cos(2\varphi)$$

$$u_\varphi = -\frac{\partial \psi}{\partial r} = -2B_0 r - \frac{C_0}{r} - D_0\cancel{(2r\ln r + r)} + \left(-2A_2 r + 2B_2 \frac{1}{r^3} \cancel{- 4C_2 r^3}\right)\cos(2\varphi)$$

$$u_\varphi = -2B_0 r - \frac{C_0}{r} + \left(-2A_2 r + 2\frac{B_2}{r^3}\right)\cos(2\varphi)$$

BC: At $r \to \infty$

$$u_\varphi \to -\frac{1}{2}\Omega r\left(1 - \cos(2\varphi)\right)$$

const:

$$-2B_0 = -\frac{1}{2}\Omega$$

$$\underline{\underline{B_0 = \frac{1}{4}\Omega}}$$

$\cos(2\varphi)$:

$$-2A_2 = +\frac{1}{2}\Omega$$

$$\underline{\underline{A_2 = -\frac{1}{4}\Omega}}$$

$u_\varphi(r=a) = 0 \quad \forall\varphi$

$$\underbrace{\left(-2B_0 a - \frac{C_0}{a}\right)}_{=0} + \underbrace{\left(-2A_2 a + 2\frac{B_2}{a^3}\right)}_{=0}\cos(2\varphi) = 0$$

$$C_0 = -2B_0 a^2 \qquad B_2 = A_2 a^4$$

$$\underline{\underline{C_0 = -\frac{1}{2}a^2\Omega}} \qquad \underline{\underline{B_2 = -\frac{1}{4}a^4\Omega}}$$

$$u_r = \frac{1}{r}\frac{\partial \psi}{\partial \varphi} = \left(-2A_2 r - 2B_2 \frac{1}{r^3} - 2\frac{D_2}{r}\right)\sin(2\varphi)$$

BC: As $r \to \infty$: $u_r \to \frac{1}{2}\Omega r \sin(2\varphi)$

$$-2A_2 = \frac{1}{2}\Omega$$

$$A_2 = -\frac{1}{4}\Omega \quad (\text{again})$$

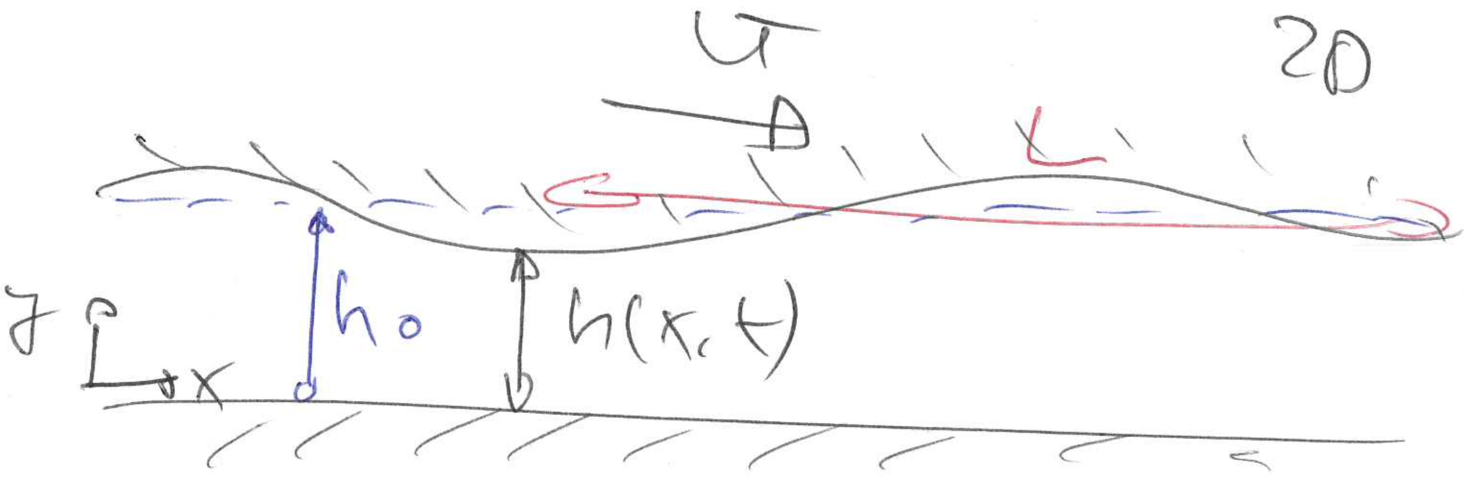
$$u_r(r=a) = 0 = \left(-2A_2 a - 2B_2 \frac{1}{a^3} - 2\frac{D_2}{a}\right)\sin(2\varphi)$$

$\vdots$

$$D_2 = \frac{1}{2}a^2\Omega \,.$$

Further example of scaling:

Lubrication theory



Gap is narrow & slowly varying

$$h_0 \ll L$$

$$\left|\frac{\partial h}{\partial x}\right| = O\left(\frac{h_0}{L}\right) \ll 1$$

$$x = L\tilde{x} \qquad t = \frac{L}{U}\tilde{t}$$

$$y = h_0\tilde{y} \qquad p = P\tilde{p}$$

$$u = U\tilde{u}$$

$$v = V\tilde{v}$$

$$\frac{U}{L}\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{V}{h_0}\frac{\partial \tilde{v}}{\partial \tilde{y}} = 0$$

These terms balance if

$$\boxed{V = U\underbrace{\left(\frac{h_0}{L}\right)}_{\ll 1}}$$

makes sense!

into x-comp. of mom. eqns:

$$\rho\left(\frac{U^2}{L}\frac{\partial \tilde{u}}{\partial \tilde{t}} + \frac{U^2}{L}\tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{U\cancel{h_0}}{\cancel{h_0}L}\tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}}\right) =$$

$$= -\frac{P}{L}\frac{\partial \tilde{p}}{\partial \tilde{x}} + \mu\left(\frac{U}{L^2}\frac{\partial^2 \tilde{u}}{\partial \tilde{x}^2} + \frac{U}{h_0^2}\frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2}\right)$$

$$\frac{\rho U^2}{L}\frac{D\tilde{u}}{D\tilde{t}} = -\frac{P}{L}\frac{\partial \tilde{p}}{\partial \tilde{x}} + \frac{\mu U}{h_0^2}\left(\underbrace{\cancel{\left(\frac{h_0}{L}\right)^2\frac{\partial^2 \tilde{u}}{\partial \tilde{x}^2}}}_{\text{very small } \left(\frac{h_0}{L}\right) \ll 1} + \frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2}\right)$$

$$\Big| \cdot \frac{h_0^2}{\mu U}$$

$$\cancel{\underbrace{\frac{\rho U h_0}{\mu}}_{Re}\underbrace{\left(\frac{h_0}{L}\right)}_{\ll 1}\frac{D\tilde{u}}{D\tilde{t}}} = -\frac{P}{\left(\frac{\mu U}{h_0}\right)}\left(\frac{h_0}{L}\right)+\frac{\partial^2\tilde{u}}{\partial\tilde{y}^2}$$

Assume $Re\left(\frac{h_0}{L}\right) \ll 1$