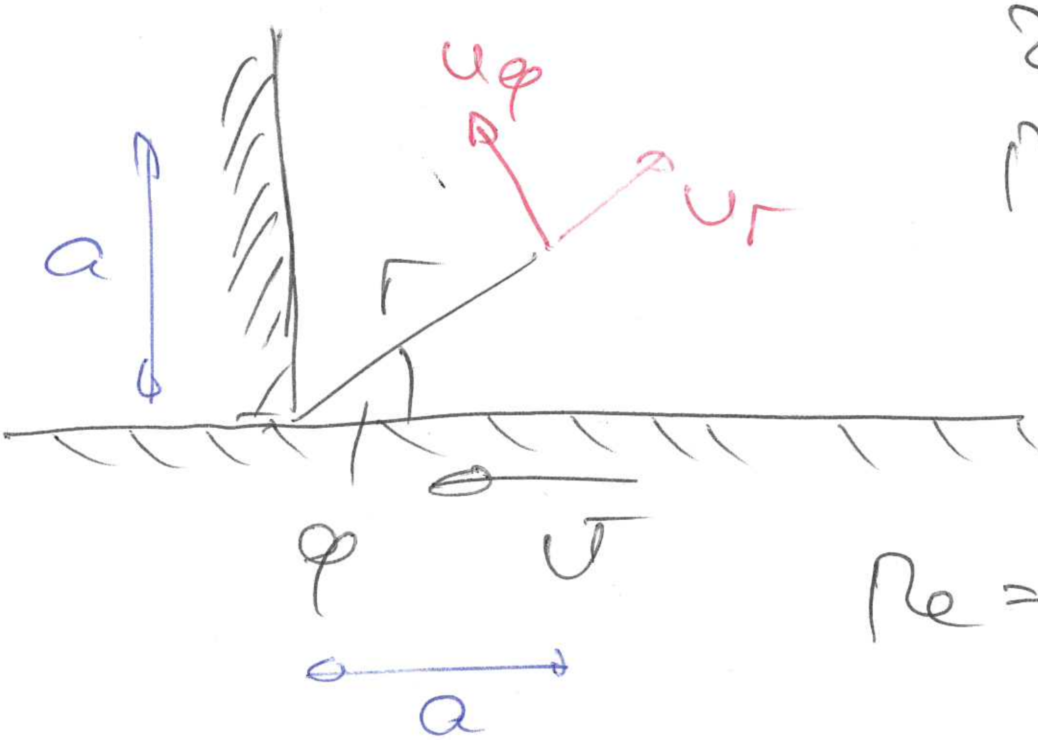
(1)




2D

$Re \ll 1$

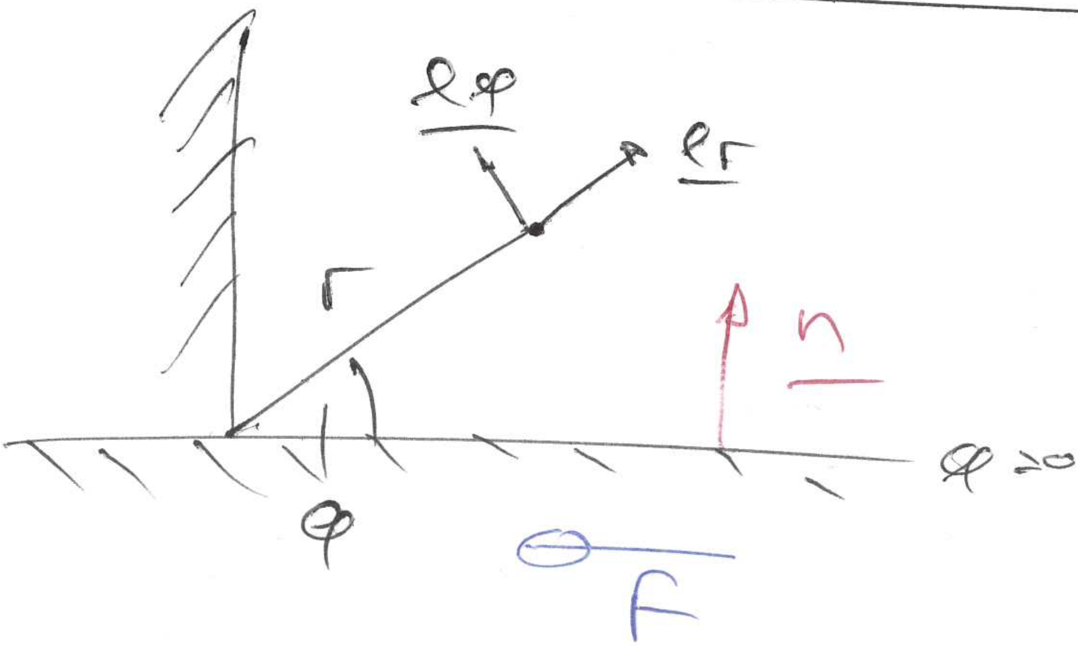
$$Re = \frac{U a \rho}{\mu}$$

$$u_r = u = f_r(\varphi)$$

$$u_\varphi = v = f_\varphi(\varphi)$$

$\underline{u}$ does $\underline{\text{not}}$ depend on $r$!

## (II) Force on bottom wall




F = tangential force on wall

Exercise:

$$F = \left| \int_0^a \tau_{r\varphi} \, dr \right| \quad @ \varphi = 0$$

$$\tau_{ij} = -p\delta_{ij} + 2\mu e_{ij} \qquad i,j = r,\varphi$$

$$\tau_{r\varphi} = 2\mu e_{r\varphi}$$

$$\frac{\tau_{r\varphi}}{\mu} = r\frac{\partial}{\partial r}\left(\frac{v}{r}\right) + \frac{1}{r}\frac{\partial u}{\partial \varphi}$$

Recall $u, v$ are indep. of $r$.

$$r\frac{\partial}{\partial r}\left(\frac{v}{r}\right) \sim r\,r^{-2} \sim \frac{1}{r}$$

$$\frac{1}{r}\frac{\partial u}{\partial \varphi} \sim \frac{1}{r}$$

$$\Rightarrow \tau_{r\varphi} \sim \frac{1}{r}$$

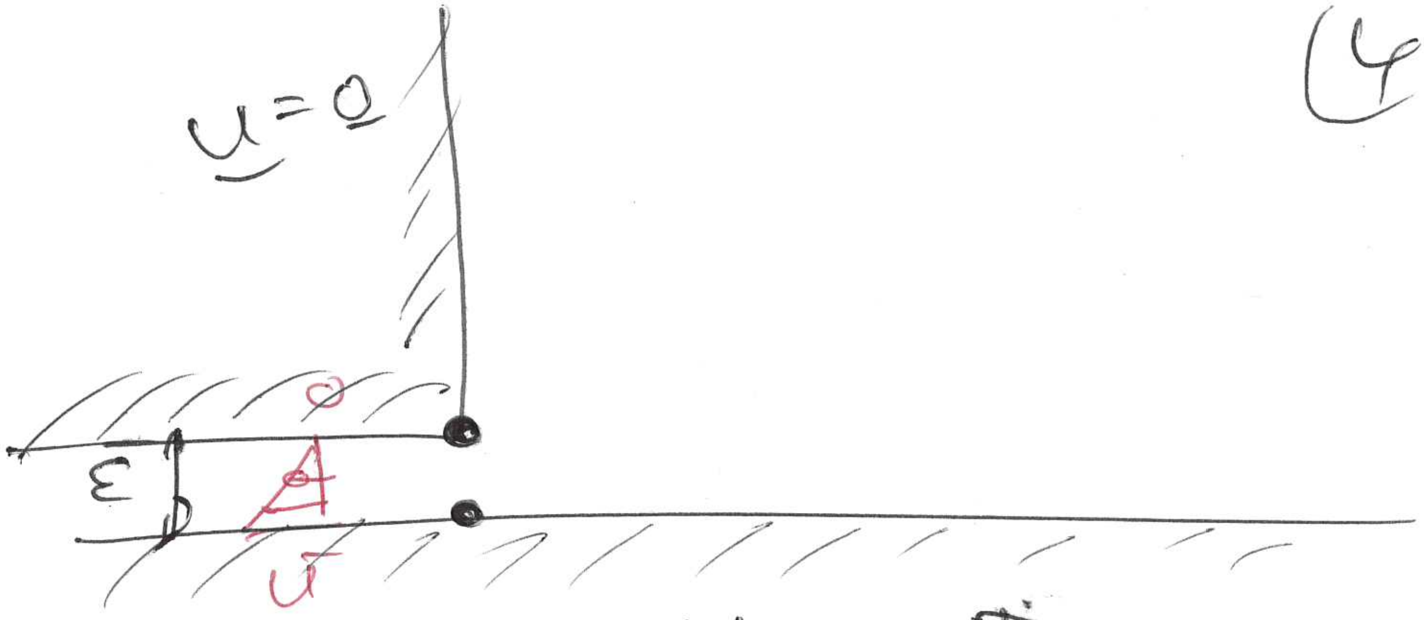
$$F = \left| \int_0^\infty \underbrace{\tau_{r\varphi}}_{\frac{1}{r}} \, dr \right| \to \infty$$

$\Rightarrow$ Force on plate is infinite!

Partly because the plate is $\infty$ large.

$$F_L = \left| \int_0^L \underbrace{\tau_{r\varphi}}_{\frac{1}{r}} \, dr \right| \text{ still } \infty$$

$\Rightarrow$ problem arises at $r = 0$




$\underline{u} = -U\underline{e_x}$

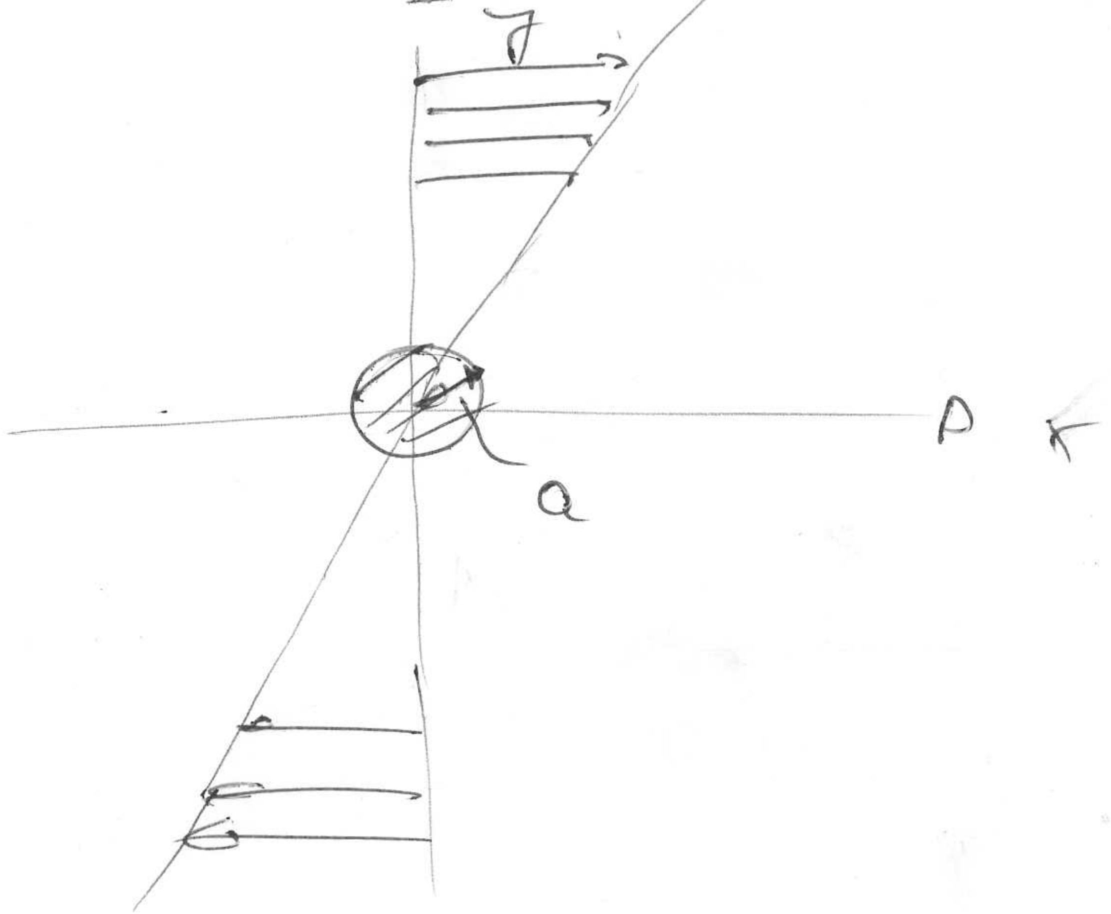
$\tau_{r\varphi} = \mu \frac{U}{\varepsilon}$

as $\varepsilon \to 0$ soln at $r = 0$ is supposed to have 2 different values.

$\tau_{r\varphi} \to \infty$ as $\varepsilon \to 0$

Can reformulate the problem by considering finite gap width $\varepsilon$.

# Example: Cylinder in shear flow




- no slip at $r = a$
- in far field:

$$\underline{u} \to \Omega y \, \underline{e}_x \quad \text{as } r \to \infty.$$

$$x = r\cos\varphi$$

$$y = r\sin\varphi$$

$$\underline{e}_x = \underline{e}_r \cos\varphi - \underline{e}_\varphi \sin\varphi$$

$$\underline{e}_y = \underline{e}_r \sin\varphi + \underline{e}_\varphi \cos\varphi$$

2nd BC becomes as $r \to \infty$

$$\underline{u} = \Omega r \sin\varphi \left( \underline{e_r} \cos\varphi - \underline{e_\varphi} \sin\varphi \right)$$

$$= \underline{e_r}\, \Omega r \underbrace{\sin\varphi \cos\varphi}_{\frac{1}{2}\sin(2\varphi)} +$$

$$+ \underline{e_\varphi} \left( -\Omega r \underbrace{\sin^2\varphi}_{\frac{1}{2}(1-\cos(2\varphi))} \right)$$

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As $r \to \infty$

$$u_r \to \frac{1}{2} \Omega r \sin(2\varphi)$$

$$u_\varphi \to -\frac{1}{2} \Omega r \left( 1 - \cos(2\varphi) \right)$$

$$\nabla^4 \psi = 0$$

$$u_r = \frac{1}{r} \frac{\partial \psi}{\partial \varphi} \qquad u_\varphi = -\frac{\partial \psi}{\partial r}$$

suggests

$$\psi(r, \varphi) = F(r) + G(r) \cos(2\varphi)$$

<u>Note:</u> we know the general soln of $\nabla^4 \psi = 0$ in cyl. polars. <u>Plan:</u> choose appropriate functions from this general soln (handout)

$$\psi = \underbrace{A_0 + B_0 r^2 + C_0 \ln r + D_0 r^2 \ln r}_{f(r)} +$$

$$\underbrace{\left(A_2 r^2 + B_2 \frac{1}{r^2} + C_2 r^4 + D_2\right)}_{G(r)} \cos(2\varphi)$$