Fluid mechanics

3 steps: I Describe mathematically the flow field / motion of fluid particles. (Kinematics)

II Formulate the equations of motion: balance of forces acting on fluid particles: stress.

III Constitutive eqns relate kinematics to stresses.

} The Navier Stokes eqns.

+ plenty of examples!

§ (n+1) Kinematics

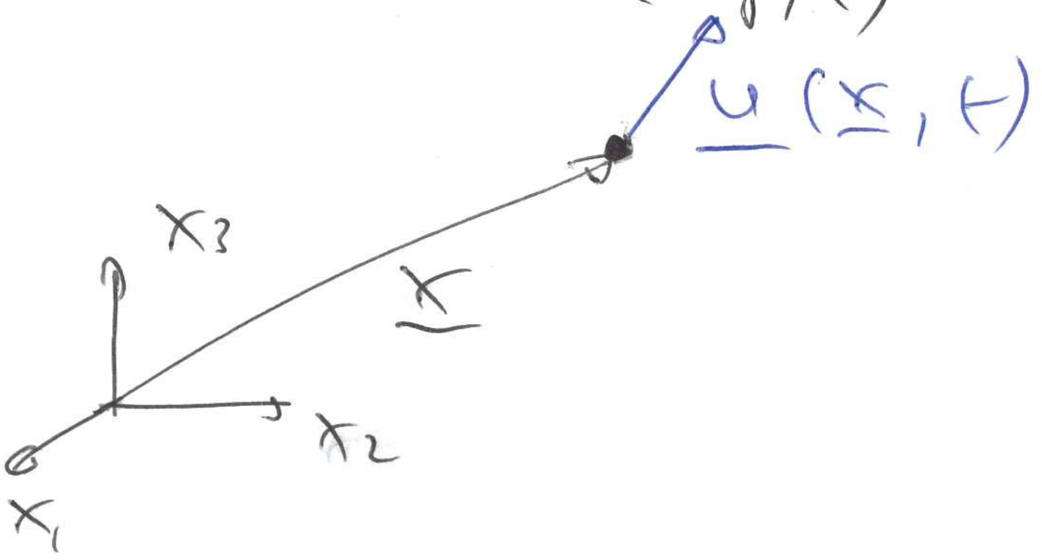
The Eulerian velocity field.

Assume we know the velocity $\underline{u}$ as a fct. of the 3 cartesian coords $(x, y, z) = (x_1, x_2, x_3)$ & time t.

$$\underline{u} = \underline{u}(x_1, x_2, x_3, t) = \underline{u}(\underline{x}, t)$$

or

$$u_i = u_i(x_j, t)$$



Note: At time t the material particle at point $\underline{x}$ has veloc. $\underline{u}$

Implies: At different times, different particles will be at this position.

This is important!

Example: Acceleration of fluid ~~particle~~ particles:

The material derivative:

The position $\underline{x}$ of a particle is given by

$$\underline{x} = \underline{x}^p(t) = \begin{pmatrix} x_1^p(t) \\ x_2^p(t) \\ x_3^p(t) \end{pmatrix}$$

= particle path

So the veloc of that particle is given by

$$\underline{u} = \underline{u}\left(x_1^p(t), x_2^p(t), x_3^p(t), t\right)$$

Accel. of this particle

$$\frac{d\underline{u}}{dt} = \frac{\partial \underline{u}}{\partial t} + \left.\frac{\partial \underline{u}}{\partial x_1}\right|_p \frac{\partial x_1^p}{\partial t} + \left.\frac{\partial \underline{u}}{\partial x_2}\right|_p \frac{\partial x_2^p}{\partial t} + \left.\frac{\partial \underline{u}}{\partial x_3}\right|_p \frac{\partial x_3^p}{\partial t}$$

index notation:

$$\frac{du_i}{dt} = \frac{\partial u_i}{\partial t} + \left.\frac{\partial u_i}{\partial x_j}\right|_p \underbrace{\frac{\partial x_j^p}{\partial t}}_{u_j}$$

$$\boxed{\frac{du_i}{dt} = \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j}}$$

Symbolically:

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + \underline{u} \cdot \nabla \underline{u}$$

often written as

$$\frac{D\underline{u}}{Dt}$$

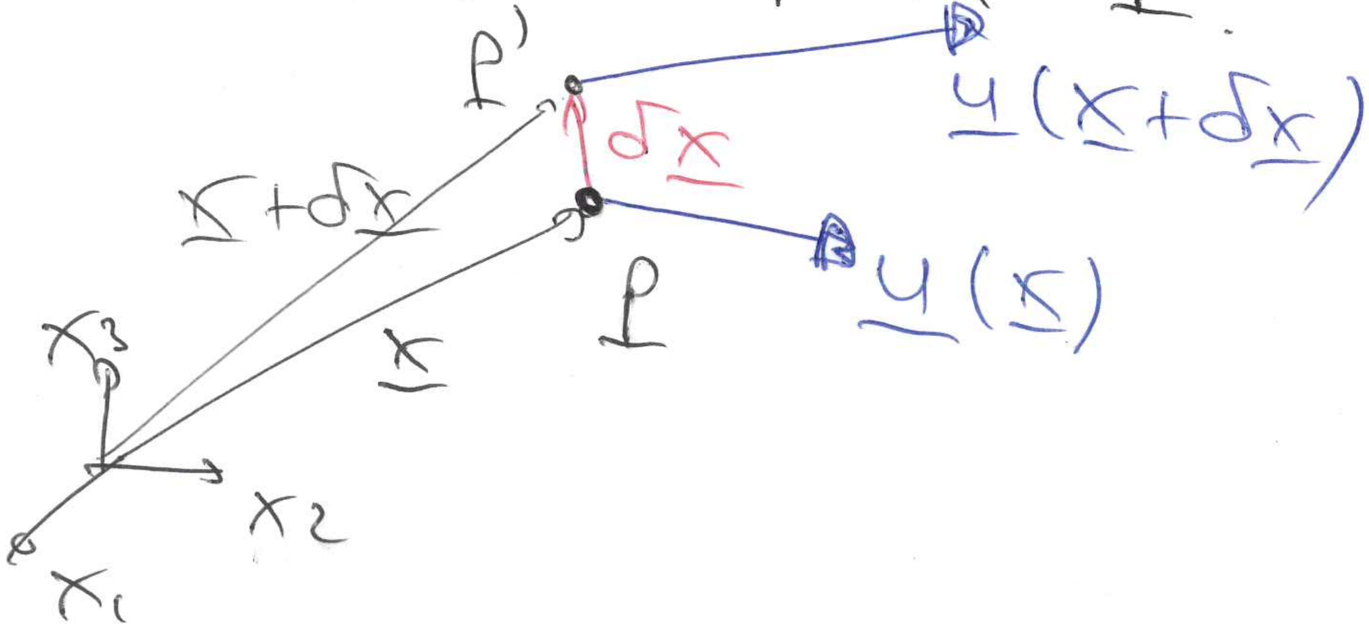
to distinuish accel. from the deriv. of the veloc. of a fixed spatial posn. $\left(\frac{\partial u}{\partial t}\right)$.

The rate of strain tensor & the vorticity

Velocity field conteins:

- translation
- rotation
- shearing
- dilation

To demonstrate this examine the veloc. field in the vicinity of a fixed spatial point P.



$$\underline{u}(\underline{x}+\delta\underline{x}) = u(x_1+\delta x_1, x_2+\delta x_2, x_3+\delta x_3)$$

3D Taylor expansion

$$= \underline{u}(x_1,x_2,x_3) + \frac{\partial \underline{u}}{\partial x_1}\delta x_1 + \frac{\partial \underline{u}}{\partial x_2}\delta x_2 + \frac{\partial \underline{u}}{\partial x_3}\delta x_3 + O(\delta \underline{x}^2)$$

$$\delta\underline{u} = \underline{u}(\underline{x}+\delta\underline{x}) - \underline{u}(\underline{x})$$

$$\delta u_i = \frac{\partial u_i}{\partial x_j}\delta x_j + O(\delta x_k^2)$$

As $\delta x_j \to 0$

$$du_i = \underbrace{\frac{\partial u_i}{\partial x_j}}_{\text{velocity gradient tensor (3×3 matrix)}} dx_j$$

Note: If $\frac{\partial u_i}{\partial x_j} = 0$

then $du_i = 0$

$\Rightarrow$ velor. does not depend on space $\Rightarrow$ all fluid particles have the same velor.

$\Rightarrow$ translation.