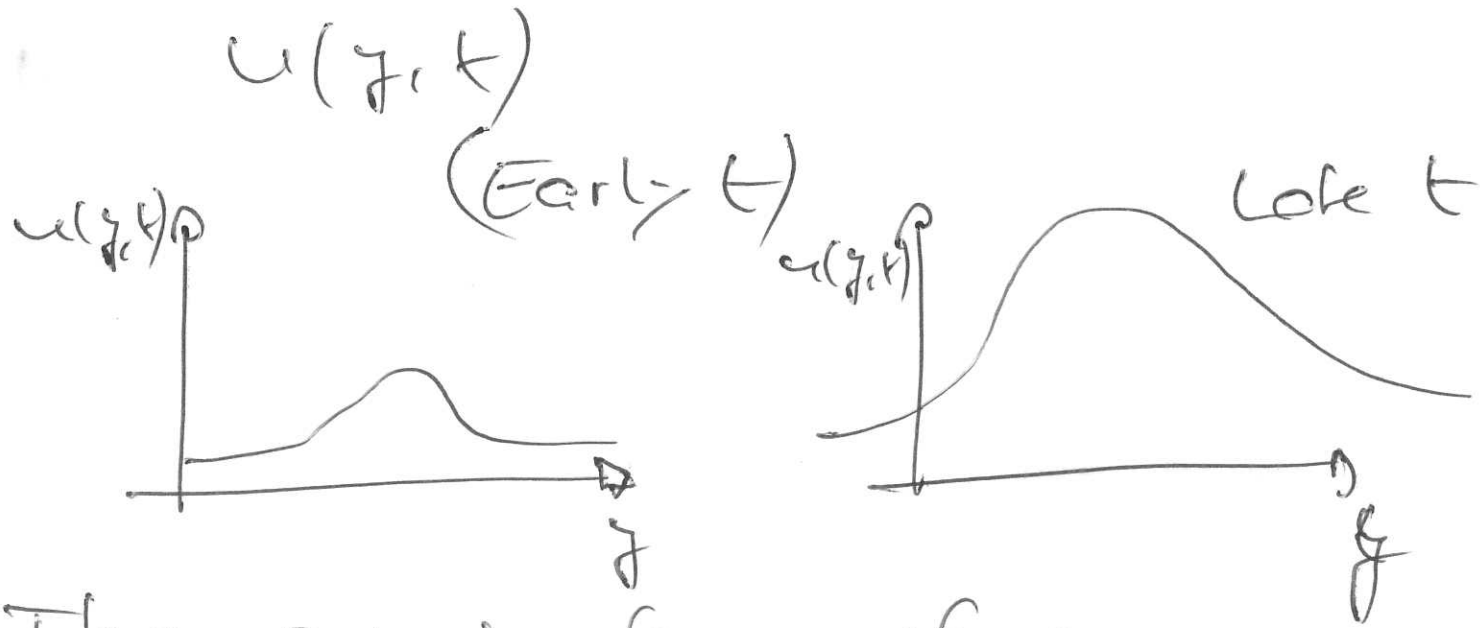
Similarity solutions

often solutions are "self-similar", i.e. they have the same "shape" but possibly a different scale at different times.



The generic form of such solns is

$$u(y,t) = a(t)\, f\left(\frac{y}{b(t)}\right)$$

↑ amplitude

↑ this scales the width of the pattern described by f.

Note: Sim. solns don't always exist - try it....

They often reduce PDEs to ODEs in a so-called similarity variable for $f(\eta)$

where $\eta = \frac{y}{b(t)}$.

The existence of a Sim. soln. is often suggested by dimensional arguments.

Example: Rayleigh's jerked plate

for $t > 0$:

parallel flow $u(y,t)$

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2}$$

IC: $u(y, t=0) = 0$

BC: $u(y=0, t) = U$ for $t > 0$

(no slip) (*)

$u \to 0$ as $y \to \infty$.

Observation:

(1) All eqns in the box are homogeneous in u apart from (*)

(2) All the eqns are linear in u.

For all eqns for which (1) & (2) apply any multiple of a given soln. is also a

soln!

⇒ Solution must be linear in U_∞

$$u(y,t;\nu,U_\infty) = U_\infty f(y,t;\nu)$$

Now: U_∞ has dimensions m/sec. $u(y,t)$ has the same dimensions.

⇒ f must be a dimensionless fct. of its arguments.

Check dimensions: (SI units)

$[y] = m$

$[t] = sec$

$[\nu] = \frac{m^2}{sec}$

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2}$$

$$[\nu] = \frac{\left[\frac{\partial u}{\partial t}\right]}{\left[\frac{\partial^2 u}{\partial y^2}\right]}$$

$$= \frac{\left(\frac{m}{sec\ sec}\right)}{\left(\frac{m}{sec\ m^2}\right)} = \frac{m^2}{sec}$$

The (only) nondimensional combination of these quantities is some power of

$$\eta = \frac{y}{\sqrt{\nu t}}$$

This is dimensionless & linear in y, which is the form we wanted for our similarity solution:

$$u(y,t;\nu,U) = U f(\eta)$$

where $\eta = \frac{y}{\sqrt{\nu t}}$

$\sqrt{\nu t}$ to $b(t)$

NEXT: insert into eqns & crank.