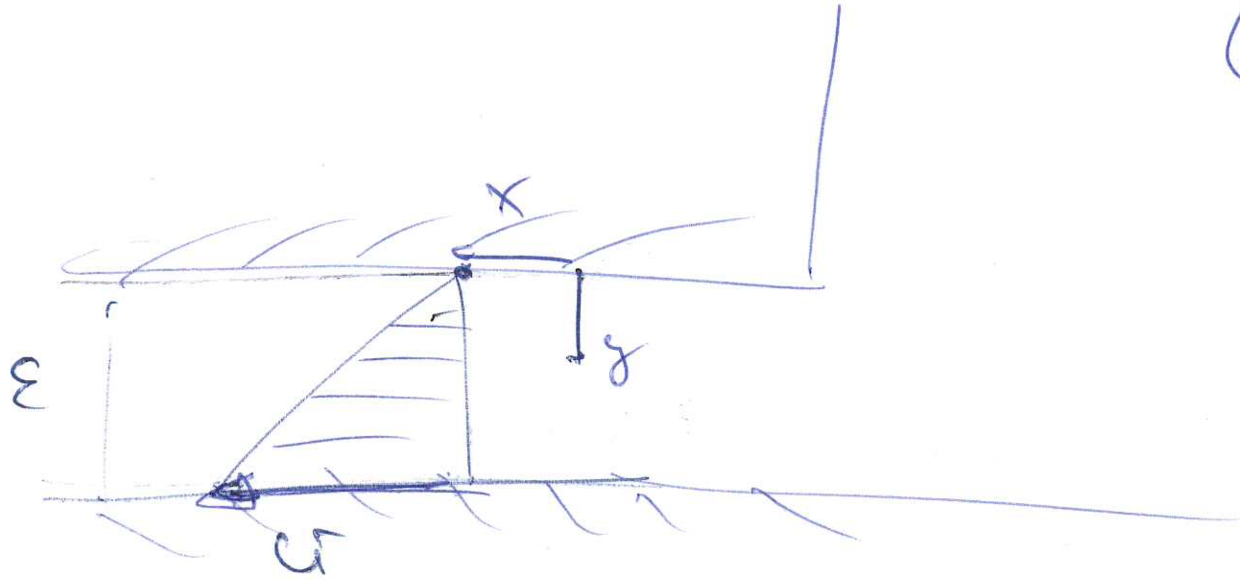
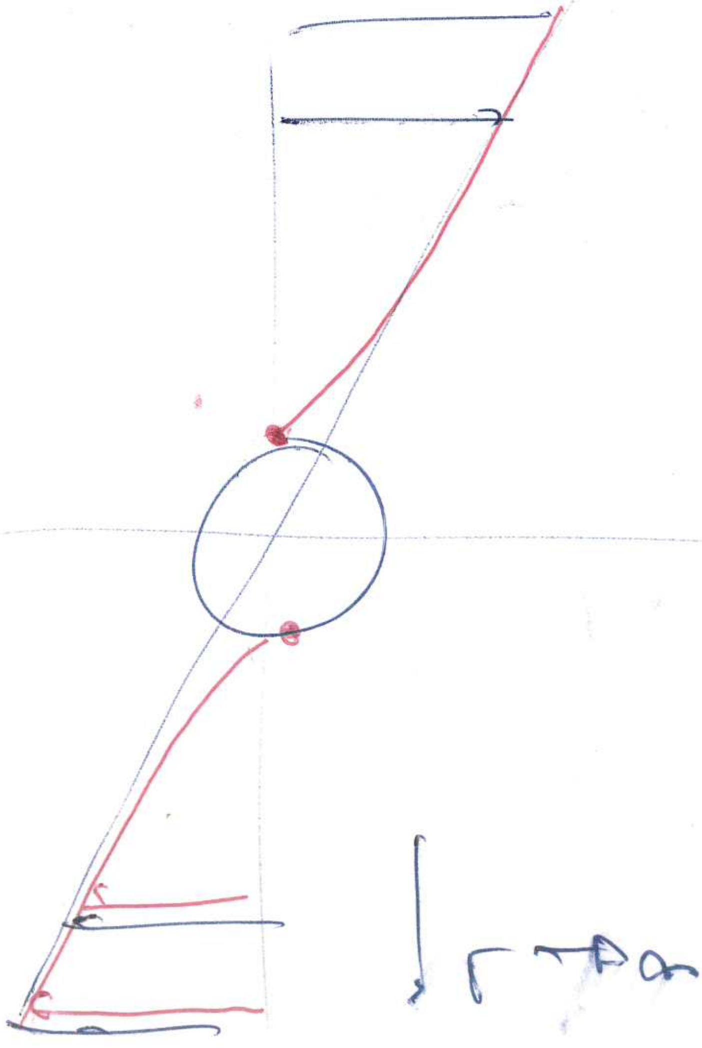


$$u(y) = U\frac{y}{\varepsilon}$$

$$\tau_{xy} = \mu\frac{\partial u}{\partial y} = \mu\frac{U}{\varepsilon}$$



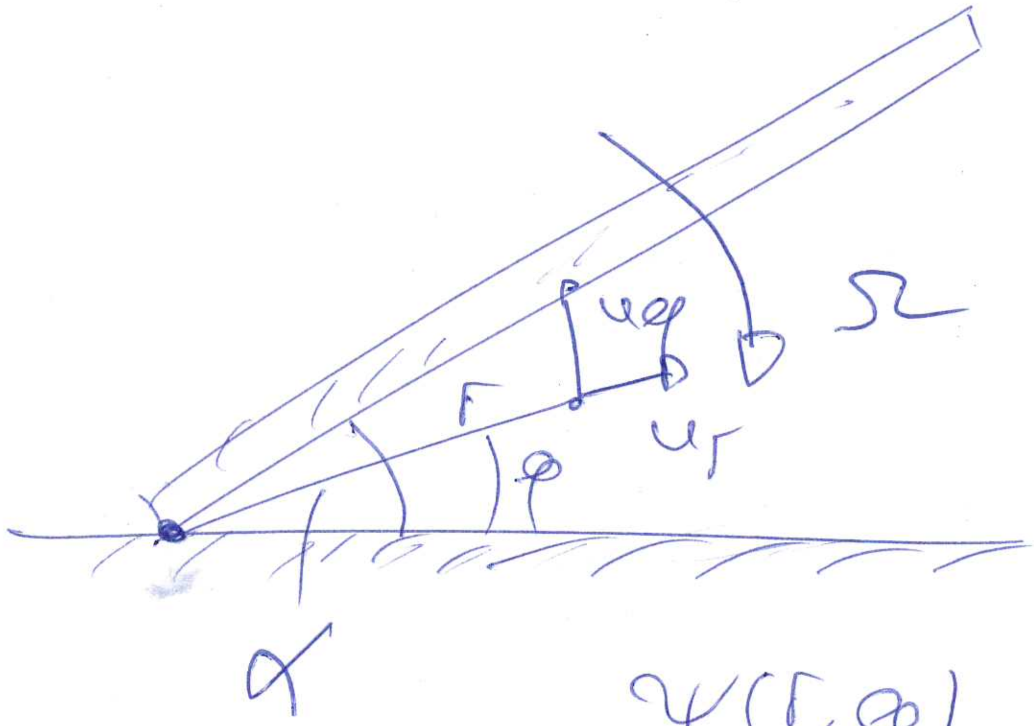
2

$$\boxed{\begin{aligned} 0 &= -\nabla p + \mu \nabla^2 \underline{u} \\ \nabla \cdot \underline{u} &= 0 \end{aligned}}$$

2D $\quad u = \frac{\partial \psi}{\partial y} \quad v = -\frac{\partial \psi}{\partial x}$

$$\Rightarrow \boxed{\nabla^4 \psi = 0}$$

(3



$$\psi(r,\varphi)$$

$$\nabla^4\psi = 0 \qquad u = u_r = \frac{1}{r}\frac{\partial\psi}{\partial\varphi}$$

$$v = u_\varphi = -\frac{\partial\psi}{\partial r}$$

$\underline{\varphi = 0:}$ $\quad u_r = \frac{1}{r}\frac{\partial\psi}{\partial\varphi} = 0$

$$\forall r$$

$$u_\varphi = -\frac{\partial\psi}{\partial r} = 0 \Rightarrow$$

$\underline{\varphi = \alpha:}$ $\quad u_r = \frac{1}{r}\frac{\partial\psi}{\partial\varphi} = 0$

$$u_\varphi = -\frac{\partial\psi}{\partial r} = -\Omega r$$

$$\frac{\partial \psi}{\partial \varphi} = 0 \quad \text{at } \varphi = 0 \qquad (1)$$

$$\frac{\partial \psi}{\partial \varphi} = 0 \quad \text{at } \varphi = \alpha \qquad (2)$$

$$-\frac{\partial \psi}{\partial r} = 0 \quad \text{at } \varphi = 0 \quad \forall r$$

$$\Longrightarrow \quad \psi(\varphi = 0) = C_1$$

$$-\frac{\partial \psi}{\partial r} = -\Omega r \quad \text{at } \varphi = \alpha \quad \forall r$$

$$\psi = \frac{1}{2}\Omega r^2 + C_2$$

Constants in ψ are arbitrary

$C_1 = 0$, say.

What about C_2

No flow through pivot between plates:

$$C_2 = 0.$$

$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$ (5)

$$\boxed{\begin{aligned} &\nabla^4 \psi = 0 \\ &\frac{\partial \psi}{\partial \varphi} = 0 \quad \text{at } \varphi = 0, \alpha \\ &\psi = 0 \quad \text{at } \varphi = 0 \\ &\psi = \tfrac{1}{2}\Omega r^2 \quad \text{at } \varphi = \alpha \ (*) \end{aligned}}$$

$$\psi = \psi(r, \varphi; \alpha, \Omega)$$

~~Show:~~

$$\cancel{\psi(r,\varphi) = \Omega r^2 f(\varphi;\alpha)}$$

Lin. & homog. apart from (*)

$$\Rightarrow \quad \underbrace{\psi(r, \varphi, \alpha, \Omega)}_{\frac{m^2}{sec}} = \Omega\, g(r, \varphi; \alpha)$$

ψ	m^2/sec
r	m
φ	1
α	1
Ω	$1/sec$

$$[u] = \left[\frac{\partial \psi}{\partial y}\right]$$

$$\frac{m}{sec} = \frac{[\psi]}{m}$$

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$$\underbrace{\psi(r,\varphi;\alpha,\Omega)}_{\frac{m^2}{sec}} = \underbrace{\Omega r^2}_{\frac{m^2}{sec}} \underbrace{f(\cancel{r},\varphi;\alpha)}_{\text{nondim fct. of its args.}}$$

$$\psi(r,\varphi) = \Omega r^2 f(\varphi)$$