Parallel flows

$\underline{u} = u(\cancel{x}, y, z, t)\, \underline{e}_x$

$$\boxed{\begin{aligned} \rho \frac{\partial u}{\partial t} &= \rho f_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \quad (1) \\ \rho f_y &= \frac{\partial p}{\partial y} \quad (2) \\ \rho f_z &= \frac{\partial p}{\partial z} \quad (3) \qquad \frac{\mu}{\rho} = \nu \end{aligned}}$$

linear! Parallel flow eqns.

Note: (2) & (3) completely determine the y & z dependence of the pressure.

Special case: No body force

$\underline{f} = \underline{0}$: (2) & (3) $\Longrightarrow p = p(x,t)$

(1): $$\underbrace{\rho \frac{\partial u}{\partial t}}_{\text{fct of } y,z,t} = \underbrace{-\frac{\partial p}{\partial x}}_{\text{fct of } x,t} + \mu \underbrace{\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)}_{\text{fct of } y,z,t}$$

$u = u(y,z,t)$

$\Longrightarrow \frac{\partial p}{\partial x}$ must not depend on x.

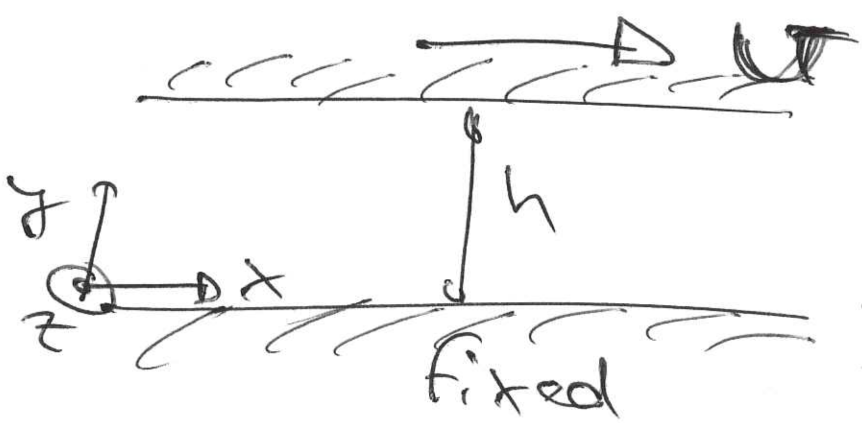
$$\frac{\partial p}{\partial x} = G(t)$$

Parallel flow eqns w/o body force

$$\boxed{\begin{aligned} \frac{\partial u}{\partial t} &= -\frac{G(t)}{\rho} + \nu\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \\ \frac{\partial p}{\partial x} &= G(t)\ ; \quad \frac{\partial p}{\partial y} = \frac{\partial p}{\partial z} = 0 \\ v &= w = 0 \end{aligned}}$$

Example: Couette flow

Flow between parallel infinite plates. Upper plate moves with veloc U.



$\underline{u} = U\,\underline{e_x}$

$\underline{u} = \underline{0}$

What do we know:

- probably

$$u(\cancel{x},y,\cancel{z},t) = u(y,t)$$

parallel flow

- probably $u(y,\cancel{t}) = u(y)$
- probably

$$G = 0.$$

no need for pressure (flow is driven by plate)

$$\cancel{\frac{\partial u}{\partial t}} = \cancel{-\frac{G}{\rho}} + \nu\left(\frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}\right)$$

$$\frac{d^2 u}{dy^2} = 0$$

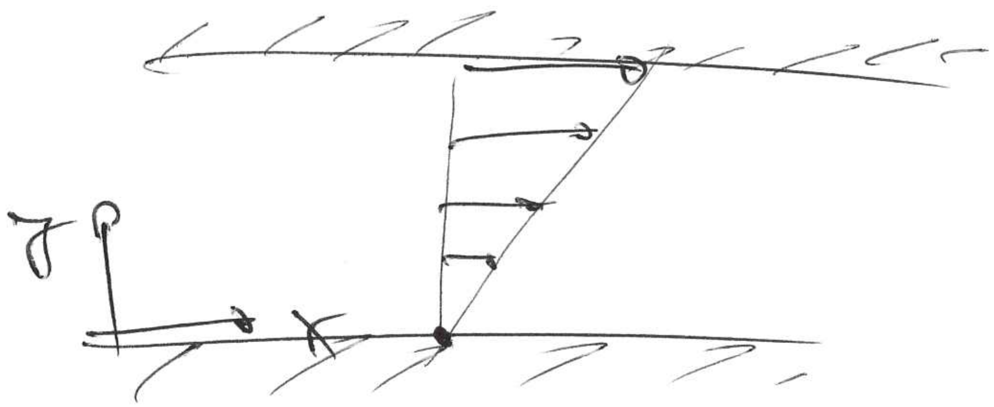
$$u(y) = Ay + B$$

<u>BC:</u> $u(y=0) = 0$

$u(y=h) = U$

$$\Rightarrow u(y) = \frac{y}{h} U$$

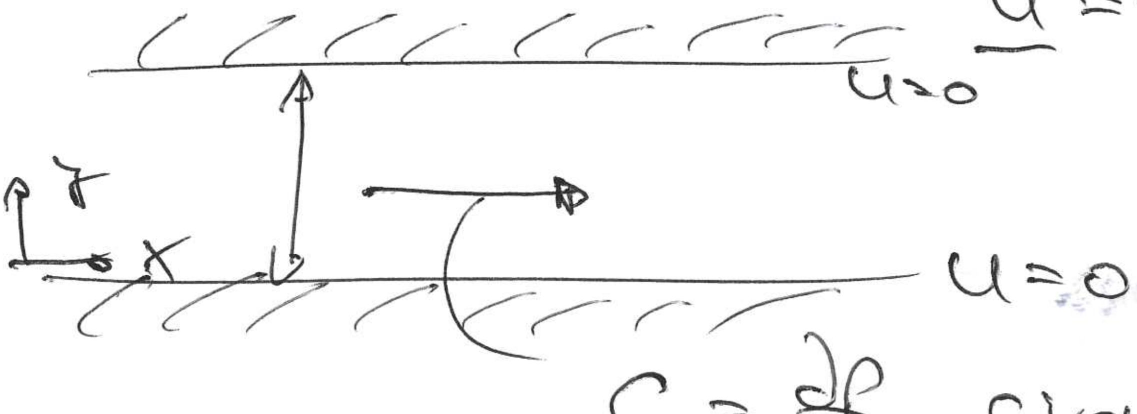
linear shear flow



Example: Poiseuille flow

pressure driven flow in a rigid channel

$\underline{u} = u(y,z,t)\, \underline{e_x}$



$G = \frac{\partial p}{\partial x}$ given.

As before:

$$u(y, \cancel{z}, \cancel{t}) = u(y)$$

$$\cancel{\rho \frac{\partial u}{\partial t}} = -G + \mu\left(\frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}\right)$$

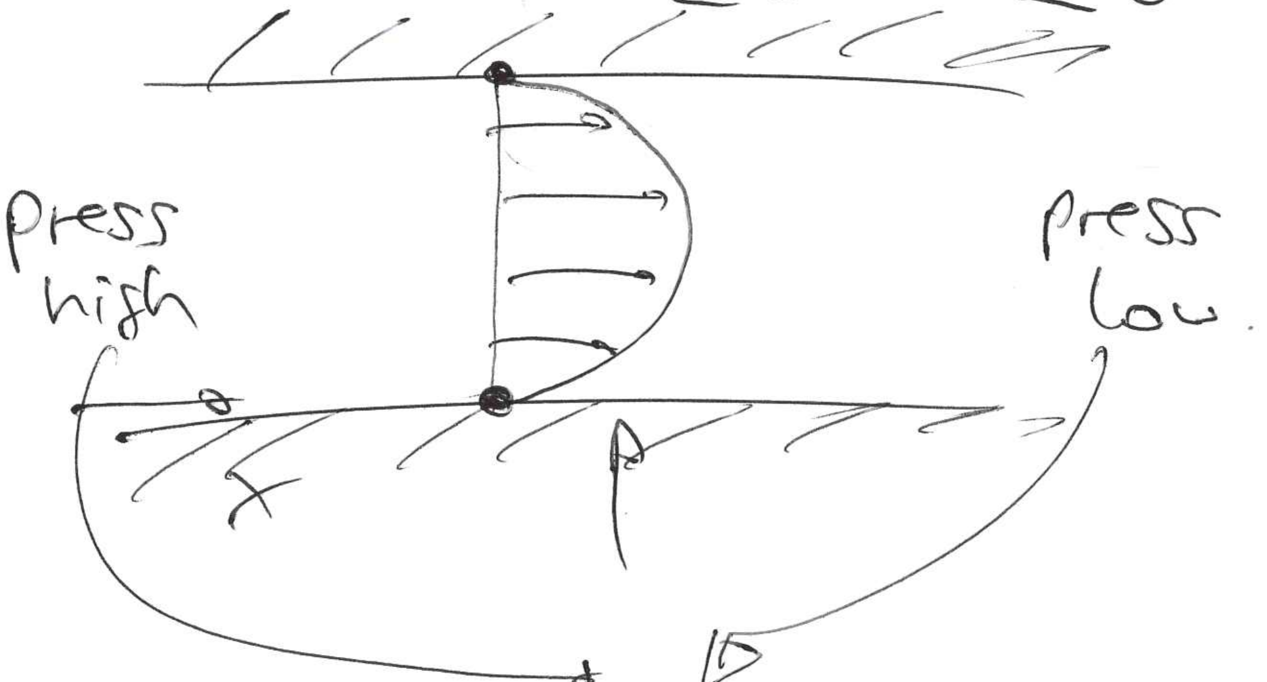
$$G = \mu \frac{d^2 u}{dy^2}$$

$$u(y) = \frac{1}{2}\frac{G}{\mu} y^2 + Ay + B$$

arb. constants

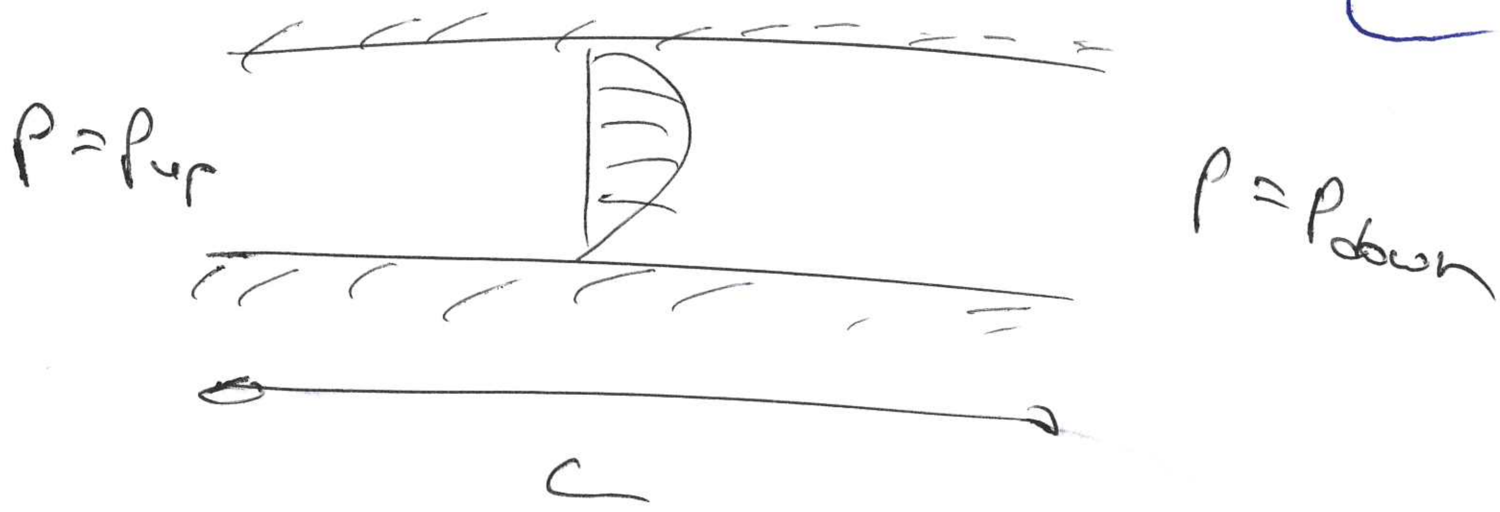
<u>2 BCs:</u> $u(y=0) = u(y=h) = 0$

$$\Rightarrow \quad u(y) = \underbrace{\frac{G}{2\mu}}_{<0}\underbrace{\left(y^2 - hy\right)}_{<0} \geq 0$$



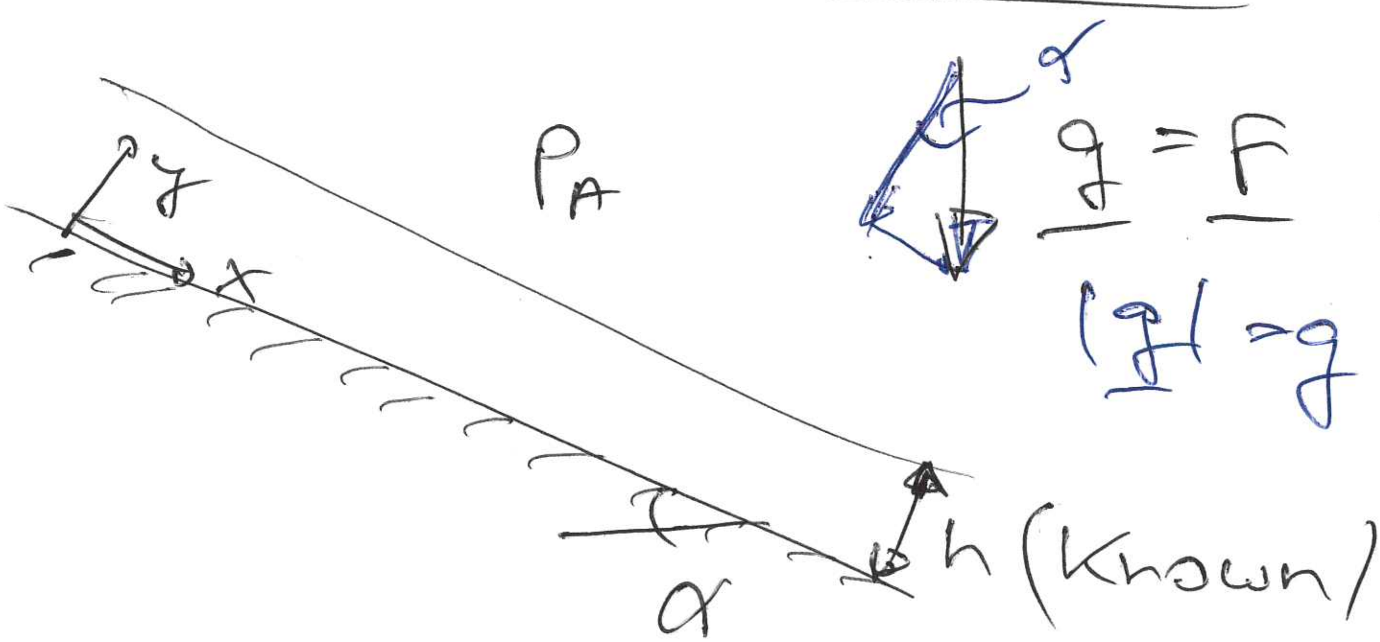
$$G = \frac{dp}{dx} < 0$$

6



$$G = \frac{p_{down} - p_{up}}{L}$$

Example: Flow down an inclined plane



$$\underline{g} = \underline{F}$$

$$|\underline{g}| = g$$

$$\underline{F} = \underline{g} = \underbrace{g \sin\alpha}_{F_x} \underline{e}_x \underbrace{- g\cos\alpha}_{F_y} \underline{e}_y$$

$$\underline{F} = f_x \, \underline{e_x} + f_y \, \underline{e_y} + f_z \, \underline{e_z}$$