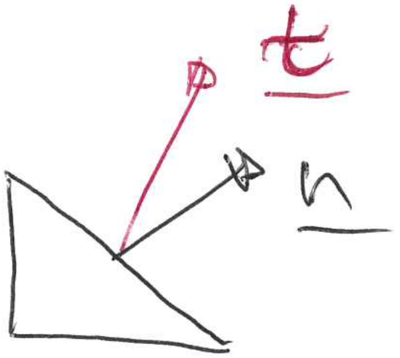
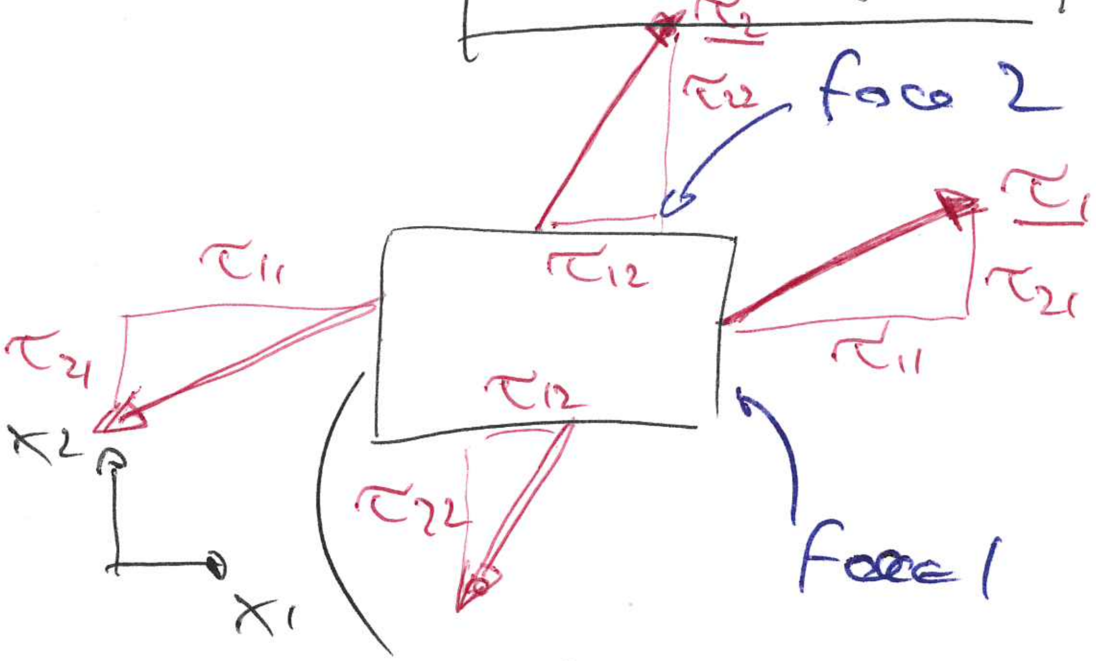


$$\boxed{t_i = \tau_{ij} n_j}$$



on this face $\underline{n} = -\underline{e}_1$

$\Rightarrow$ traction / stress points in opposite direction

Can now express stress in terms of the stress tensor τ_{ij} & orientation of a face (in terms $\underline{n}$).

Later: Determine τ_{ij} as a fct. of e_{ij}.

Particular stress states:

(i) Hydrostatic pressure

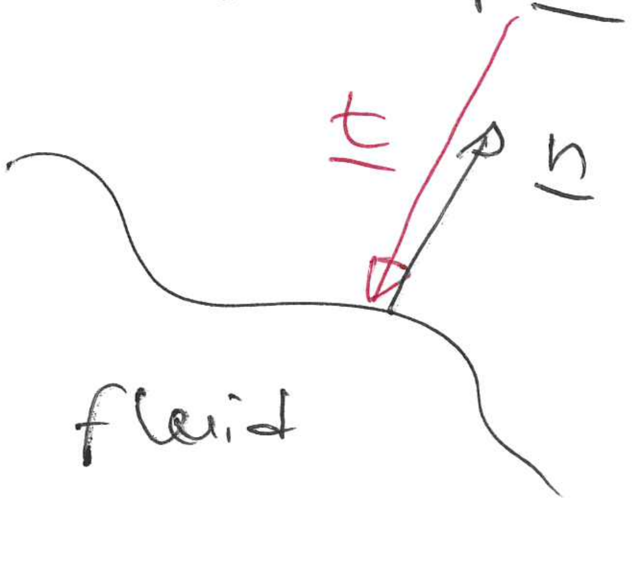
$$\tau_{ij} = -p\,\delta_{ij}$$

implies that the traction is always normal to the face & uniform in all directions:

$$t_i = \tau_{ij} n_j = -p\,\delta_{ij} n_j$$

$$t_i = -p\, n_i$$

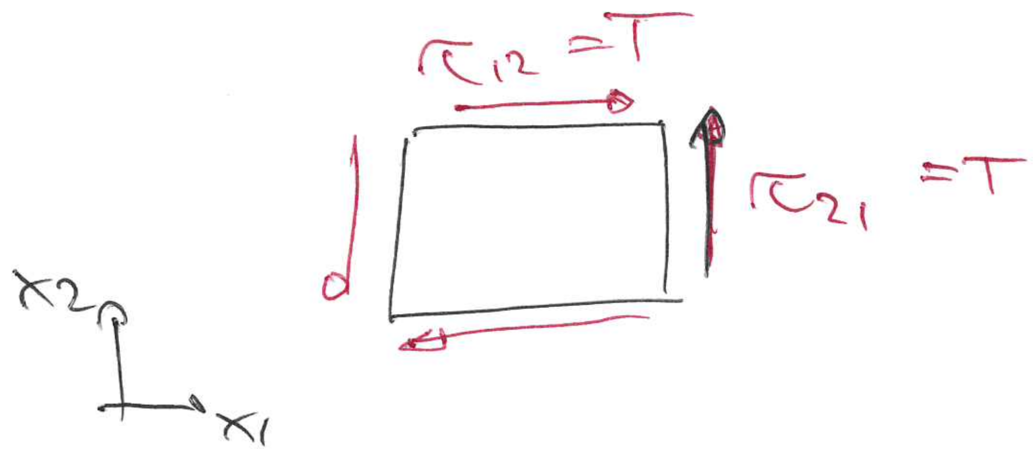
$$\underline{t} = -p\,\underline{n}$$



(ii) pure shear stress

e.g. $\tau_{12} = \tau_{21} = T$

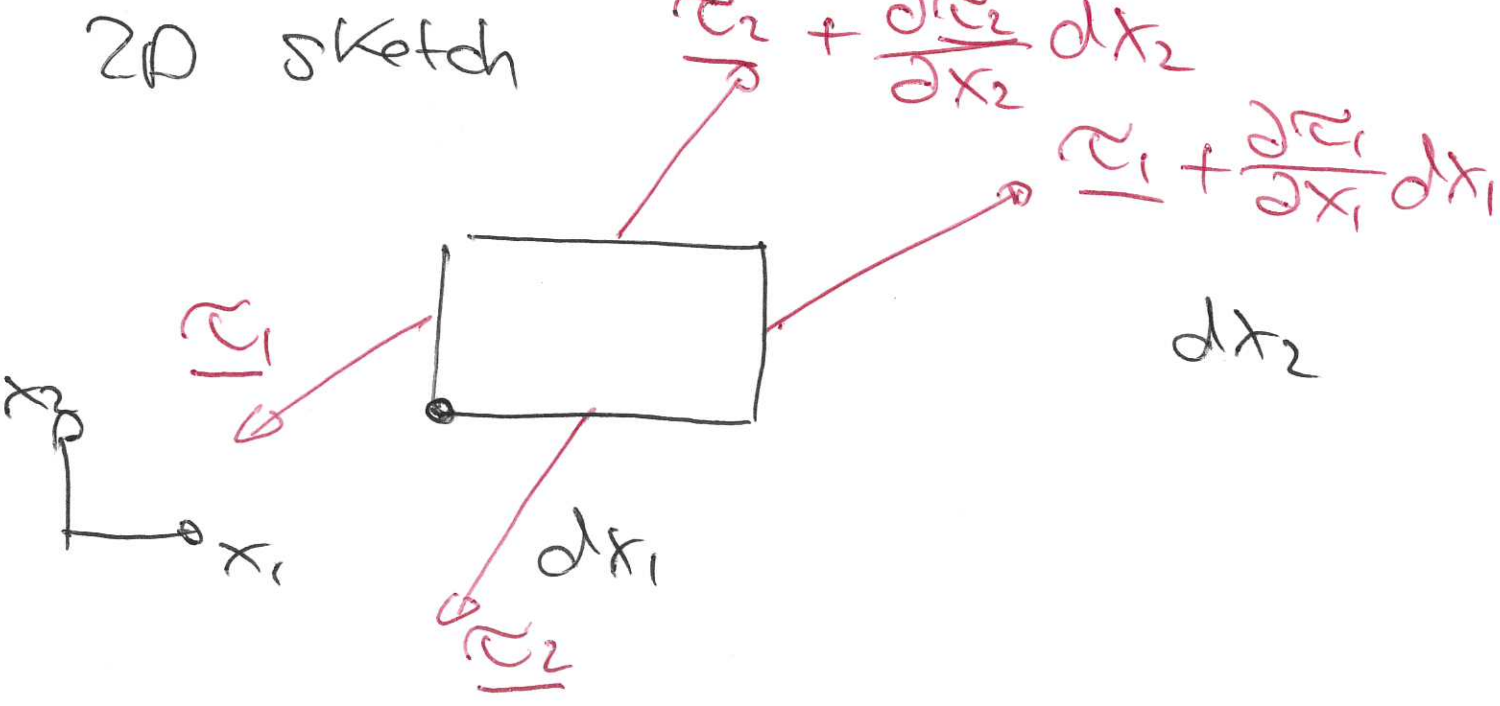
$\tau_{11} = \tau_{22} = 0$



3. Equilibrium of forces
$\Rightarrow$ Cauchy's eqn

$\sum$ forces = mass $\times$ accel.

2D sketch



Sum of all forces:

$\underline{\tau_1}$ & $\underline{\tau_2}$ cancel $\Rightarrow$ only increments remain

$$\underbrace{\frac{\partial \underline{\tau}_1}{\partial x_1} \cancel{dx_1}}_{\text{increm. in traction}} \cancel{dx_2} + \frac{\partial \underline{\tau}_2}{\partial x_2} \cancel{dx_2}\, \cancel{dx_1}$$

$$+ \underset{\text{density}}{\rho}\ \underset{\text{body force (per unit mass) e.g. gravitational accel. or centrifugal forces.}}{\underline{F}}\ \cancel{dx_1}\, \cancel{dx_2} = \rho \underset{\text{material deriv.}}{\frac{Du}{Dt}} \cancel{dx_1}\, \cancel{dx_2}$$

Cauchy's eqn.:

$$\boxed{\begin{aligned} \frac{\partial \underline{\tau}_j}{\partial x_j} + \rho \underline{F} &= \rho \frac{Du}{Dt} \\ \frac{\partial \tau_{ij}}{\partial x_j} + \rho F_i &= \rho \frac{Du_i}{Dt} \end{aligned}}$$

$$\frac{\partial \tau_{ij}}{\partial x_j} + \rho f_i = \rho\left(\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j}\right)$$

Still need τ_{ij} as a fct of $\underline{u}$. $\Longrightarrow$ constitutive eqns.

INTERLUDE:

$$\tau_{ij} = \tau_{ji}$$

Can show this by balance of moments

Constitutive eqns link the Stress & the Kinematics of flow. In general this requires experiments!

We will restrict ourselves to incompressible fluids.

Observations:

Fluids:

- Can generate hydrostatic pressures.
- have a resistance to sheer flows. (honey on knife)
- do not generate internal stresses when subjected to rigid body motions.

$\Rightarrow$ The stress tensor τ_{ij} should contain a hydrostatic pressure contribution & depend on the rate of strain tensor.

A wide range of fluids (Newtonian fluids) behave according to

$$\tau_{ij} = \underbrace{-p\,\delta_{ij}}_{\text{hydrostatics}} + 2\mu\, \underbrace{e_{ij}}_{\text{rate of strain}}$$

(dynamic) viscosity has to be measured.

$$\tau_{ij} = -p\,\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$