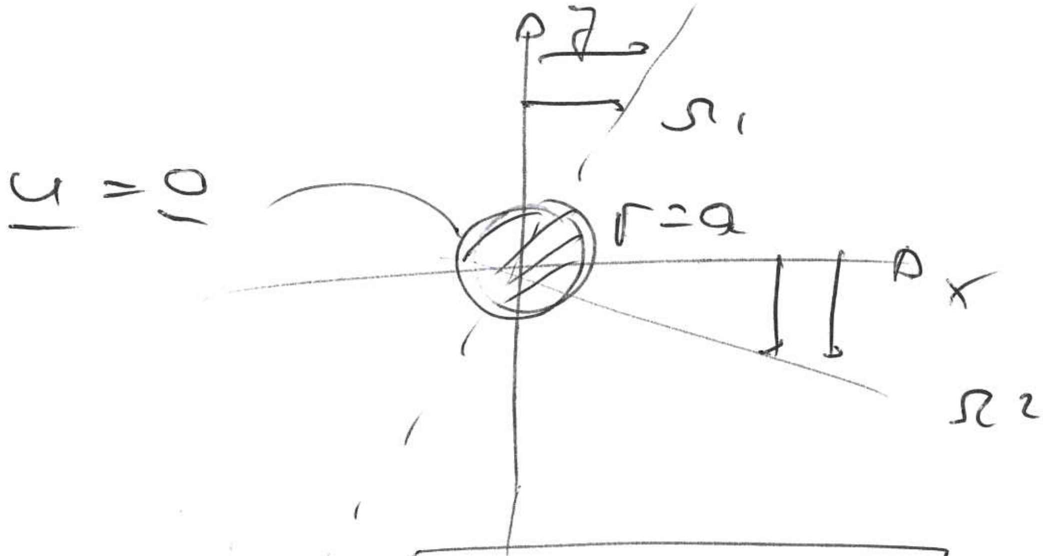

$$\boxed{\nabla^4 \psi = 0}$$

$r \to \infty$

$$u_r \to \tfrac{1}{2} r \left(\Omega_1 - \Omega_2 \right) \sin(2\varphi)$$

$$\boxed{u_\varphi \to -\tfrac{1}{2} r \left[(\Omega_1 + \Omega_2) + (\Omega_2 - \Omega_1) \cos(2\varphi) \right]}$$

$$u_r = \frac{1}{r} \frac{\partial \psi}{\partial \varphi} \qquad u_\varphi = -\frac{\partial \psi}{\partial r}$$

$\Rightarrow$ Ansatz: $\psi(r,\varphi) = f(r) + G(r) \cos(2\varphi)$

we know general soln to $\nabla^4 \psi = 0$ in cyl. polars:

$$\psi(r,\varphi) = f_0(r) + f_1(r)\cos(\varphi) + \tilde{f}_1(r) \sin(\varphi) + f_2(r)\cos(2\varphi) + \tilde{f}_2(r)\sin(2\varphi) + \ldots$$

$$\psi(r,\varphi) = f_0(r) + \sum_{n=1}^{\infty}\left(f_n(r)\cos(n\varphi) + \tilde{f}_n(r)\sin(n\varphi)\right)\ldots$$

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$f_n(r)$ & $\tilde{f}_n(r)$ are known.

So here use $f_0(r)$ & $f_2(r)$

$$\psi(r,\varphi) = \underbrace{\cancel{A_0} + B_0 r^2 + C_0 \ln r + D_0 r^2 \ln r}_{f_0(r)}$$

$$+ \underbrace{\left(A_2 r^2 + B_2 \frac{1}{r^2} + C_2 r^4 + D_2\right)}_{f_2(r)} \cos(2\varphi)$$

arb. constants in ψ are irrelevant.

$$u_\varphi = -\frac{\partial \psi}{\partial r} = -2B_0 r - \frac{C_0}{r} - \cancel{D_0\left(2r\ln r + r\right)} +$$

$$+ \left(-2A_2 r + 2B_2 \frac{1}{r^3} - \cancel{4C_2 r^3}\right)\cos(2\varphi)$$

$\to \infty$ as $r \to \infty$

grows to ∞ as $r \to \infty$
too quickly.

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BC: As $r \to \infty$: $\underline{\rho_0 = \sigma_2 = 0}$

$$u_\varphi \to -\frac{1}{2} r (\Omega_1 + \Omega_2) - \frac{1}{2} r (\Omega_2 - \Omega_1) \cos(2\varphi)$$

Match const. term (in $\cos(2\varphi)$)

$$\Rightarrow -2B_0 = -\frac{1}{2}(\Omega_1 + \Omega_2)$$

$$\underline{B_0 = \frac{1}{4}(\Omega_1 + \Omega_2)}$$

Match $\cos(2\varphi)$ terms:

$$-2A_2 = -\frac{1}{2}(\Omega_2 - \Omega_1)$$

$$\underline{A_2 = \frac{1}{4}(\Omega_2 - \Omega_1)}$$

Also: $u_\varphi = 0$ at $r = a$.

$$\underbrace{\left(-2B_0 a - \frac{\sigma_0}{a}\right)}_{=0} + \underbrace{\left(-2A_2 a + \frac{2B_2}{a^3}\right)}_{=0} \cos(2\varphi) = 0$$

$\forall \varphi$

$$\sigma_0 = -2a^2 B_0$$

$$\underline{\sigma_0 = -\frac{1}{2} a^2 (\Omega_1 + \Omega_2)}$$

$$B_2 = a^4 A_2$$

$$\underline{B_2 = \frac{1}{4} a^4 (\Omega_2 - \Omega_1)}$$

$$u_r = \frac{1}{r}\frac{\partial \psi}{\partial \varphi} = \left(-2A_2 r \cancel{+} - 2B_2 \frac{1}{r^3} - 2\frac{D_2}{r}\right)\sin(2\varphi)$$

<u>BC:</u> $r \to \infty$:

$$u_r \to \frac{1}{2} r (\Omega_1 - \Omega_2) \sin(2\varphi)$$

$$\Rightarrow \quad -2A_2 = \frac{1}{2}(\Omega_1 - \Omega_2)$$

from matching terms
that are linear in r

$$A_2 = \frac{1}{4}(\Omega_2 - \Omega_1)$$

<u>again!</u>

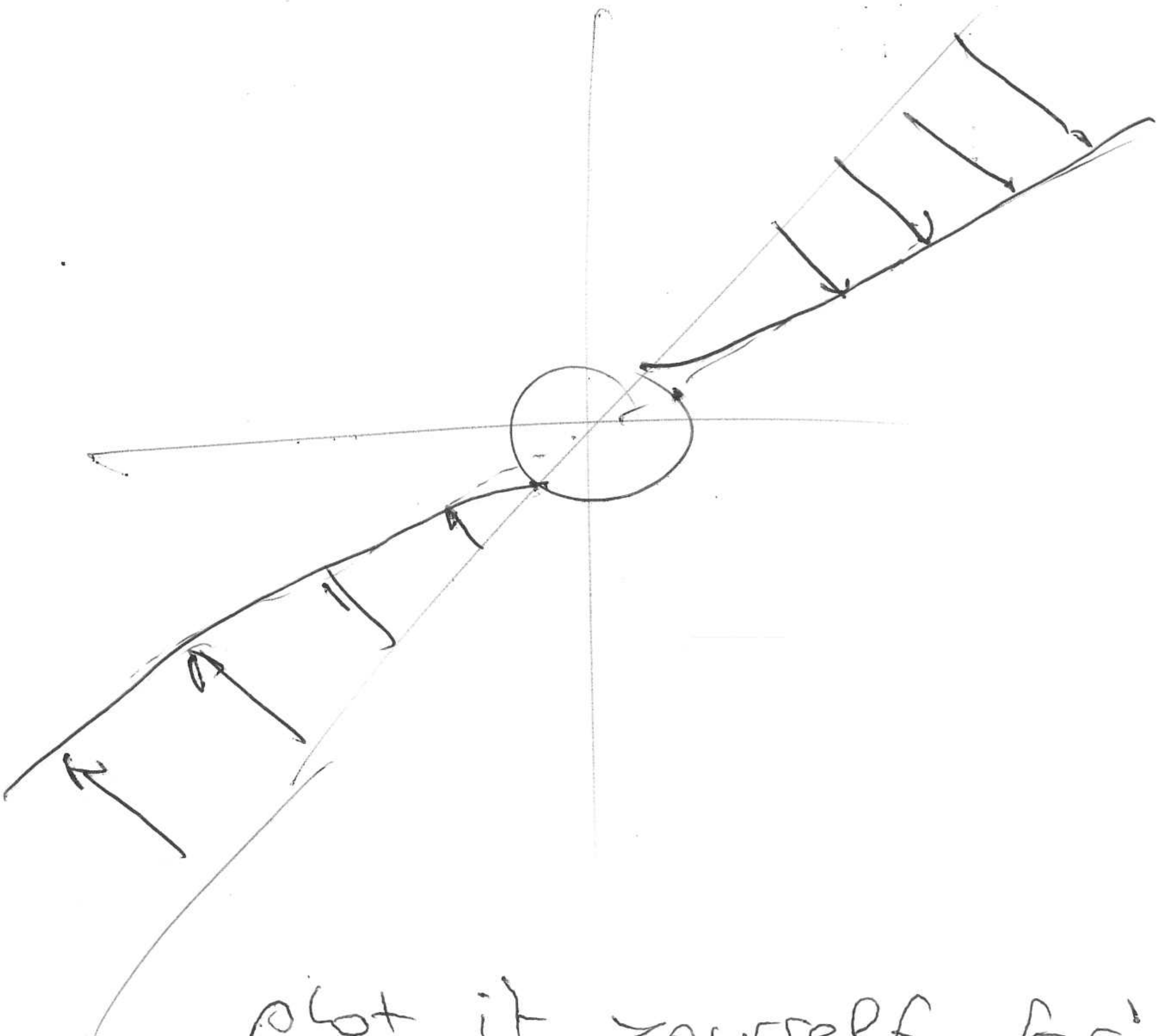
<u>Also:</u> $u_r(r = a) = 0$

$$-2A_2 a - 2B_2 \frac{1}{a^3} - 2\frac{D_2}{a} = 0$$

insert A_2 & B_2 from above

$$\underline{D_2 = \frac{1}{2} a^2 (\Omega_1 - \Omega_2)}$$

Flow field. $\Omega_2 = \Omega_1$



plot it yourself for
other values of Ω_1 & Ω_2

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