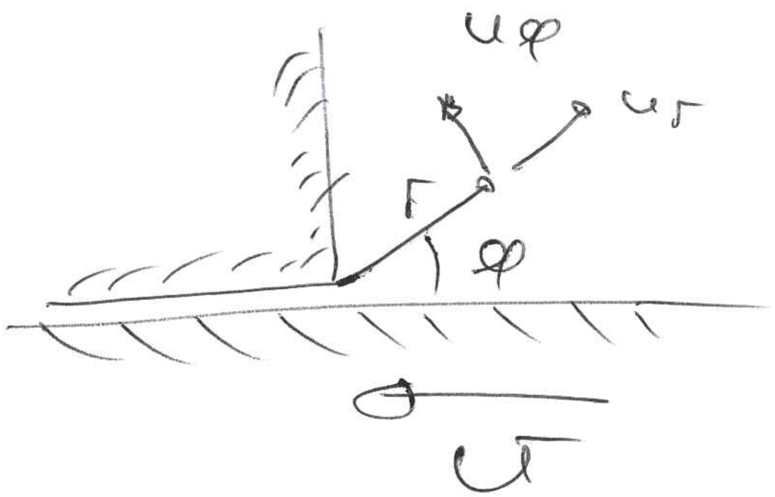


$Re \ll 1$

$$D^4\psi = 0$$

$$u_r = \frac{1}{r}\frac{\partial \psi}{\partial \varphi}$$

$$u_\varphi = -\frac{\partial \psi}{\partial r}$$

$$\psi = \frac{U r}{\left(\frac{\pi}{2}\right)^2 - 1}\left(-\left(\frac{\pi}{2}\right)^2 \sin\varphi + \varphi\cos\varphi + \frac{\pi}{2}\varphi\sin\varphi\right)$$

$u_\varphi = v = -\frac{\partial \psi}{\partial r}$ indep. of r.

$u_r = u = \frac{1}{r}\frac{\partial \psi}{\partial \varphi}$ indep of r too.

Discussion

(I) Nonuniformity of the solution

Had assumed $Re \ll 1$

density

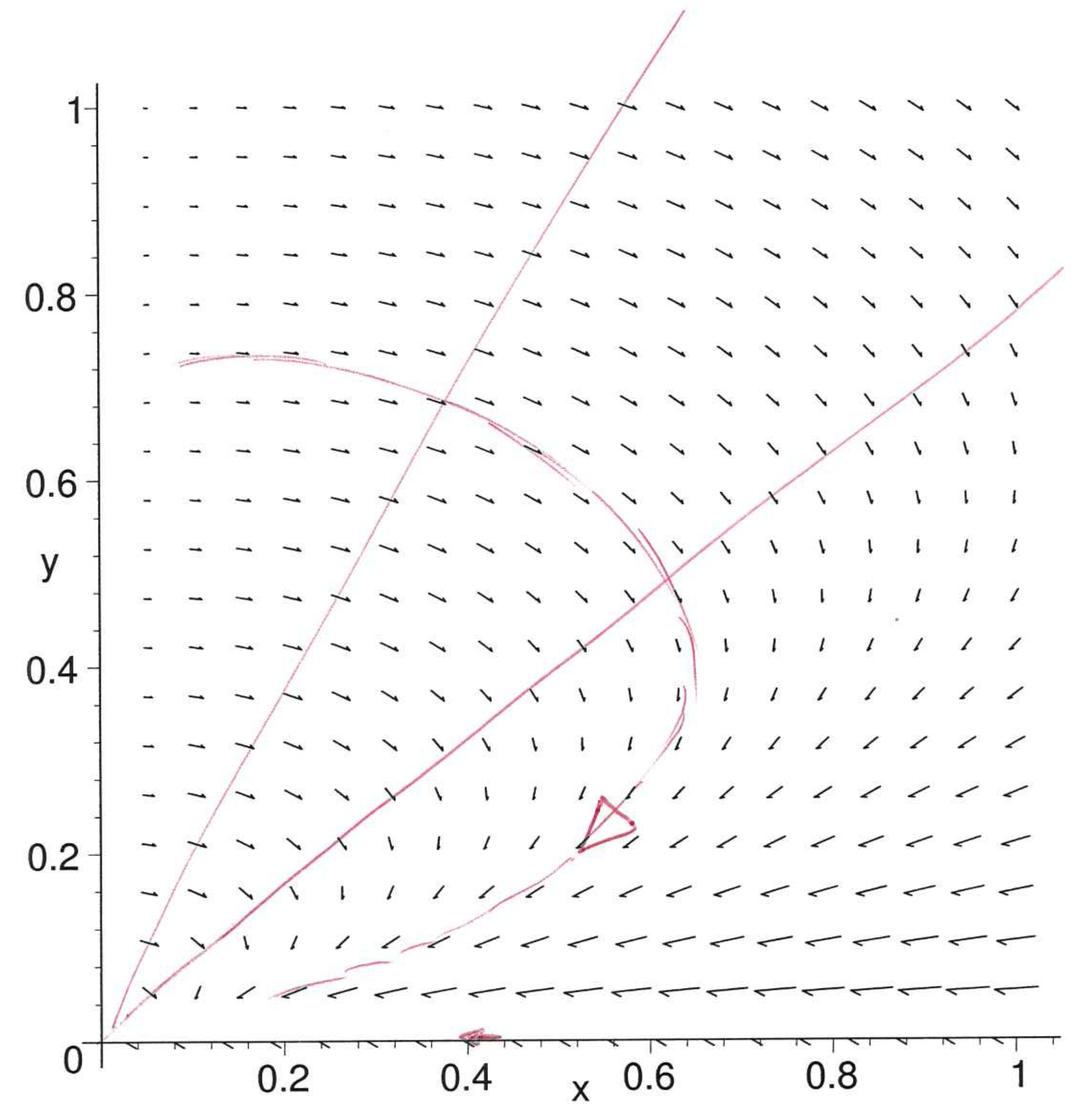
veloc scale.

$$Re = \cancel{\quad} \frac{\rho L U}{\mu}$$

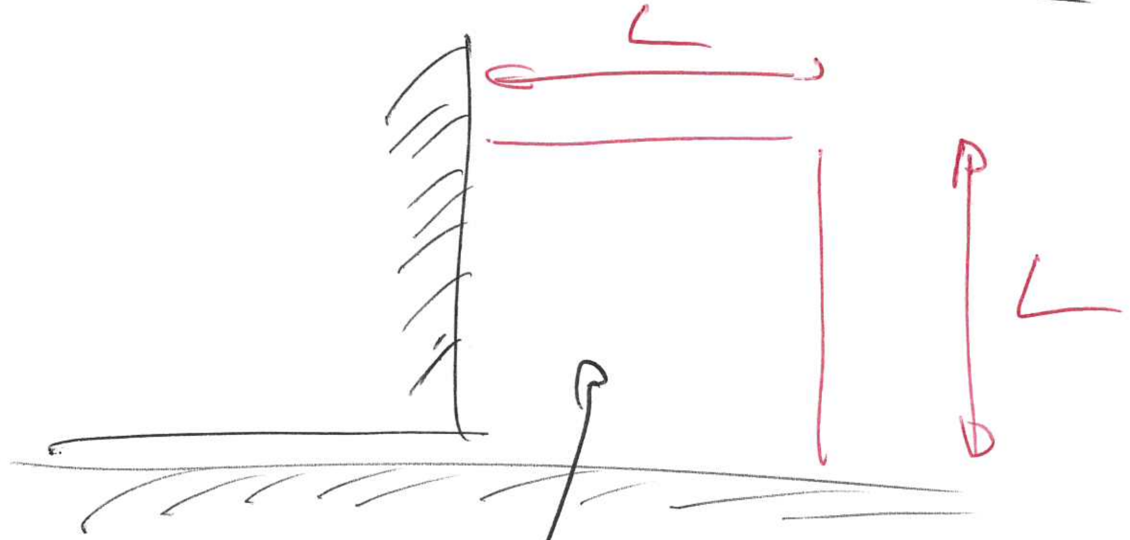
viscos.

What is L?

Velocity field for scraping flow at zero Reynolds number:
The vertical wall at x=0 is stationary. The horizontal wall at y=0
moves to the left with unit velocity.



There is no lengthscale in the problem! ⇒ we have to choose one



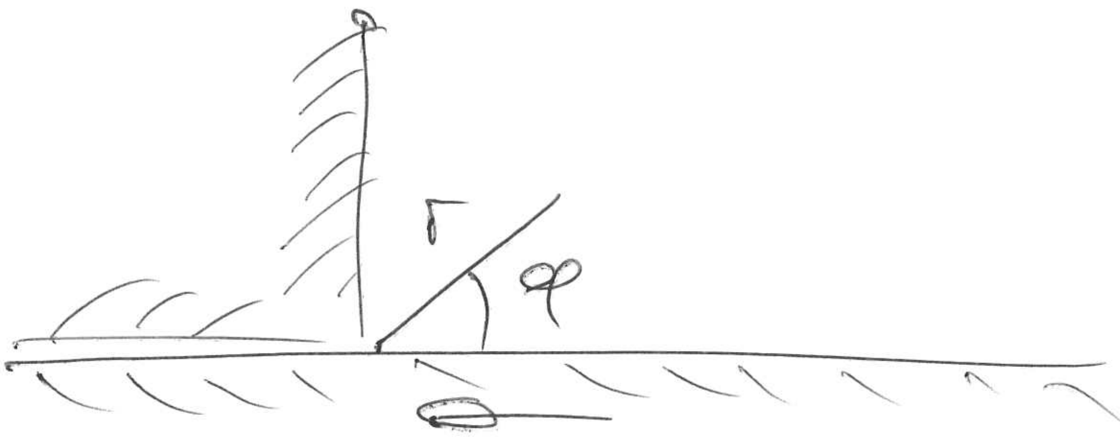
Within this region here Re is well defined & can be made as small as we wish if we take $L \to 0$.

But at large distances from corner the Reynolds number always becomes large & the Stokes eqns are not valid.

⇒ Stokes soln is only

a local approximation. (not uniformly valid everywhere).

(II) Traction on bottom wall



F ← force acting on fluid.

we're interested in tangential traction $\tau_{r\varphi}$.

$$F = \left| \int_0^\infty \tau_{r\varphi} \, dr \right|$$

$$\tau_{ij} = -p\delta_{ij} + 2\mu e_{ij}$$

so $\tau_{r\varphi} = 2\mu e_{r\varphi}$ "$i, j = r, \varphi$"

$$\frac{\tau_{r\varphi}}{\mu} = r\frac{\partial}{\partial r}\left(\frac{v}{r}\right) + \frac{1}{r}\frac{\partial u}{\partial \varphi}$$

at $\varphi = 0$

Recall neither u nor v depend on r.

$$r\frac{\partial}{\partial r}\left(\frac{v}{r}\right) = r\, r^{-2} \dots \sim \frac{1}{r}$$

$$\frac{1}{r}\frac{\partial u}{\partial \varphi} \sim \frac{1}{r}$$

$$\tau_{r\varphi} \sim \frac{1}{r}$$

$$F = \mu \int_0^\infty \frac{1}{r} \dots dr$$ is infinite!

∞ force to move bottom plate?

Due to ∞ length of bottom plate?

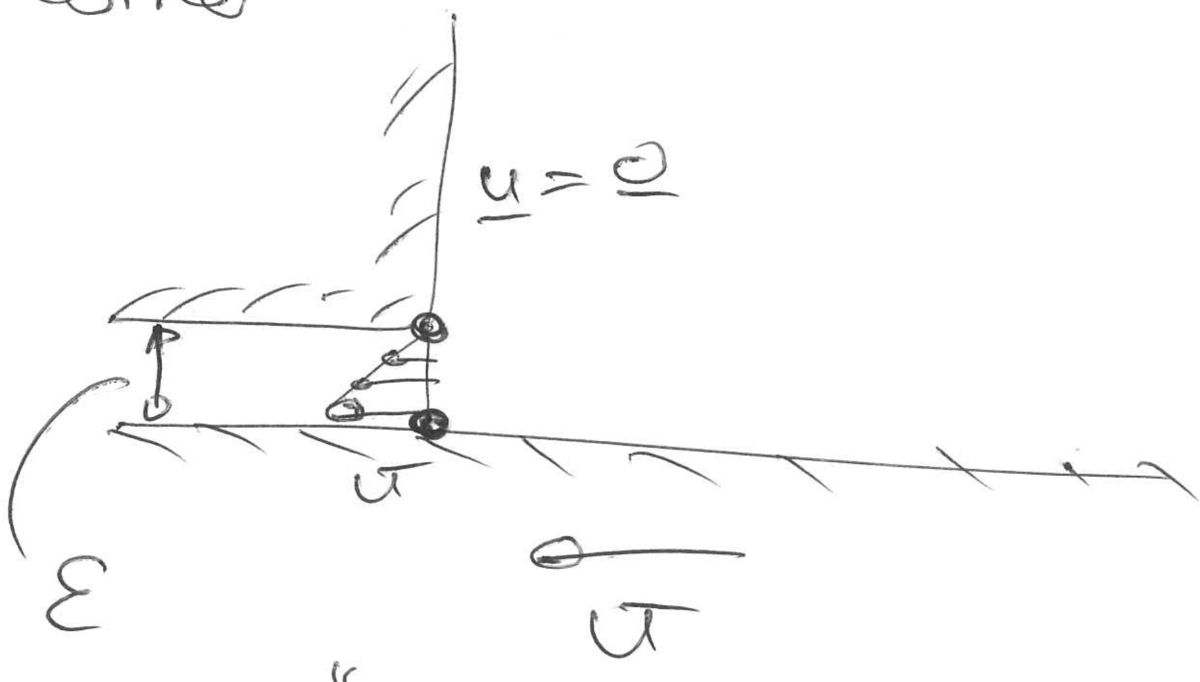
Consider force on finite segment of bottom wall ⑥

$$F_L = \mu \int_0^L \frac{1}{r} \dots dr$$

and is still ∞!

<u>Note:</u> Singularity arises at the origin.

Corner



within "gap" of width ε velocity is reduced from U to 0. $\Rightarrow$ Shear rate:

$$\cancel{e}_{xy} = e_{r\varphi} \sim \frac{U}{\varepsilon} \to \infty$$

as

$\varepsilon \to 0$.

The problem is the jump in the ~~?~~ veloc. BC in the corner. $\underline{u} = \underline{0}$ if viewed from above or $\underline{u} = -U\underline{e}_x$ if viewed from below.

$\Rightarrow$ inconsistent.

Resolution:

could incorporate $e_{r\varphi}$ into model but cannot solve this any more (analytically).

Also inconsistent from modelling point of view

On very small lengthscales (comparable to the molecular lengthscale) N. St. eqns don't apply anyway.