Fluid mechanics

3 steps:

(I) Describe (mathematically) the flow field / motion of fluid particles.

(KINEMATICS)

(II) Formulate the eqns of motion: Newton's law applied to continua. (Stresses)

(III) Constitutive eqns relate kinematics to the stresses.

} The Navier Stokes eqns.

Then: Lots of examples.

§2 Kinematics

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The Eulerian flow field

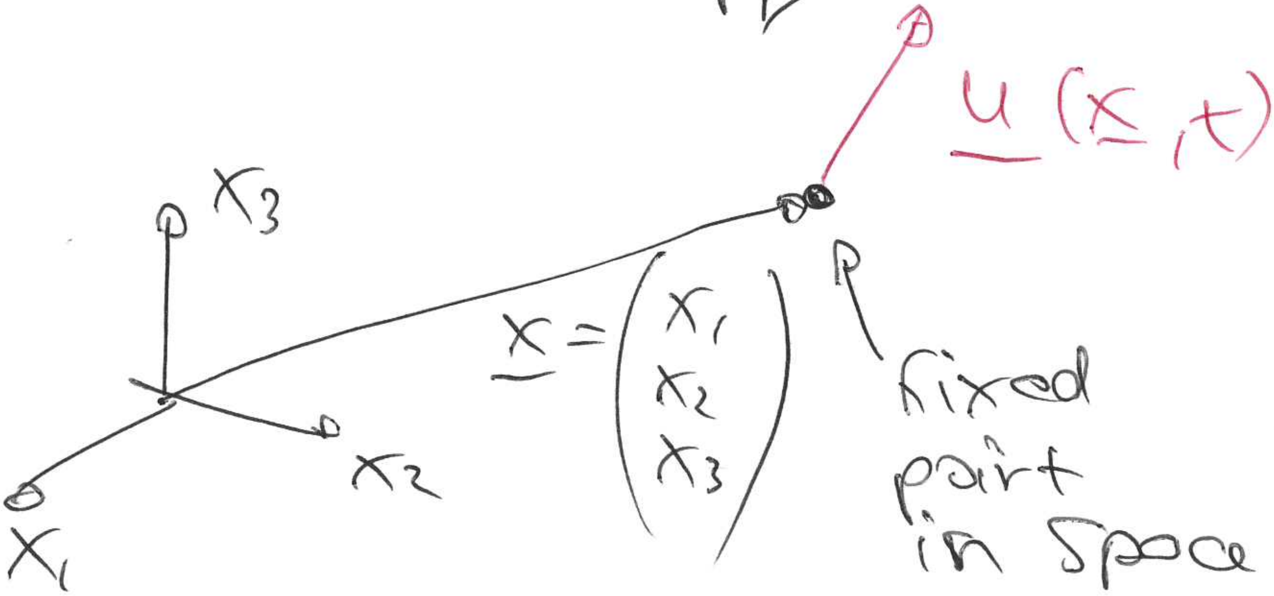
Assume we know the velocity $\underline{u}$ as a fct of the 3 cartesian coords $(x_1, x_2, x_3) = (x, y, z)$ and time t.

$$\underline{u} = \underline{u}(x_1, x_2, x_3, t) = \underline{u}(\underline{x}, t)$$

or

$$u_i = u_i(x_j, t)$$

sloppy



Note: At different times different material particles will be at point $\underline{x}$.

This is important!

E.g.: Acceleration of fluid particles.

The material derivative.

Now follow a particle whose instantaneous posn is given by the particle path:

$$\underline{x} = \underline{x}^p(t) = \begin{pmatrix} x_1^p(t) \\ x_2^p(t) \\ x_3^p(t) \end{pmatrix}$$

Its velocity is:

$$\underline{u}(x_1, x_2, x_3, t) = \underline{u}(x_1^p(t), x_2^p(t), x_3^p(t), t)$$

(4)

Accel. of the particle(!) is the rate of change of $\underline{u}$

$$\frac{d\underline{u}}{dt} = \frac{d\underline{u}(x_1^p(t), x_2^p(t), x_3^p(t), t)}{dt}$$

$$= \frac{\partial \underline{u}}{\partial t} + \left.\frac{\partial \underline{u}}{\partial x_1}\right|_{\underline{x}^p} \frac{\partial x_1^p}{\partial t} + \left.\frac{\partial \underline{u}}{\partial x_2}\right|_{\underline{x}^p} \frac{\partial x_2^p}{\partial t}$$

$$+ \left.\frac{\partial \underline{u}}{\partial x_3}\right|_{\underline{x}^p} \frac{\partial x_3^p}{\partial t}$$

$$\frac{du_i}{dt} = \frac{\partial u_i}{\partial t} + \left.\frac{\partial u_i}{\partial x_j}\right|_{\underline{x}^p} \underbrace{\frac{\partial x_j^p}{\partial t}}_{u_j}$$

$$\boxed{\frac{du_i}{dt} = \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j}}$$

Symbolic:

$$\frac{d\underline{u}}{dt} = \frac{\partial \underline{u}}{\partial t} + \underline{u} \cdot \nabla \underline{u}$$

This is often denoted by

$$\frac{D\underline{u}}{Dt}$$

The rate of strain tensor & the vorticity

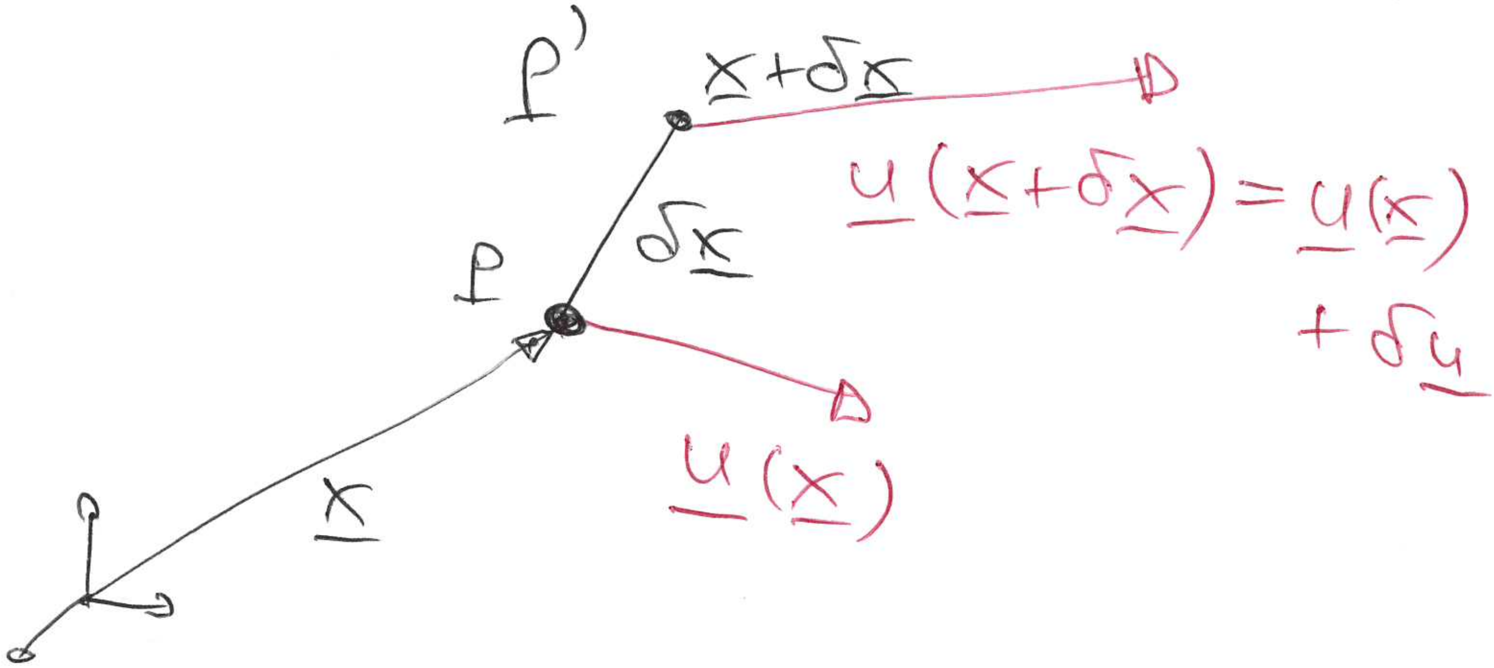
Velocity field in itself is not that interesting. It contains:

- translation
- rotation
- Shearing
- Dilation

How do we identify these?

Examine veloc. field in the vicinity of a given point P: (6

$\|\delta \underline{x}\| \ll 1$



$$\underline{u}(\underline{x}+\delta\underline{x}) = \underline{u}(x_1+\delta x_1, x_2+\delta x_2, x_3+\delta x_3)$$

Taylor expansion

$$= \underline{u}(x_1, x_2, x_3) + \frac{\partial \underline{u}}{\partial x_1}\delta x_1 + \frac{\partial \underline{u}}{\partial x_2}\delta x_2 + \frac{\partial \underline{u}}{\partial x_3}\delta x_3 + \ldots O(|\delta \underline{x}|^2)$$

$$\underbrace{\underline{u}(\underline{x}+\delta\underline{x}) - \underline{u}(\underline{x})}_{\delta\underline{u}} = \frac{\partial \underline{u}}{\partial x_j}\delta x_j + \ldots$$

Now as $|\delta \underline{x}| \rightarrow 0$

$$\delta \underline{x} \Rightarrow d\underline{x}$$

$$\boxed{du_i = \frac{\partial u_i}{\partial x_j} dx_j}$$

velocity
gradient
tensor
(3x3 matrix).