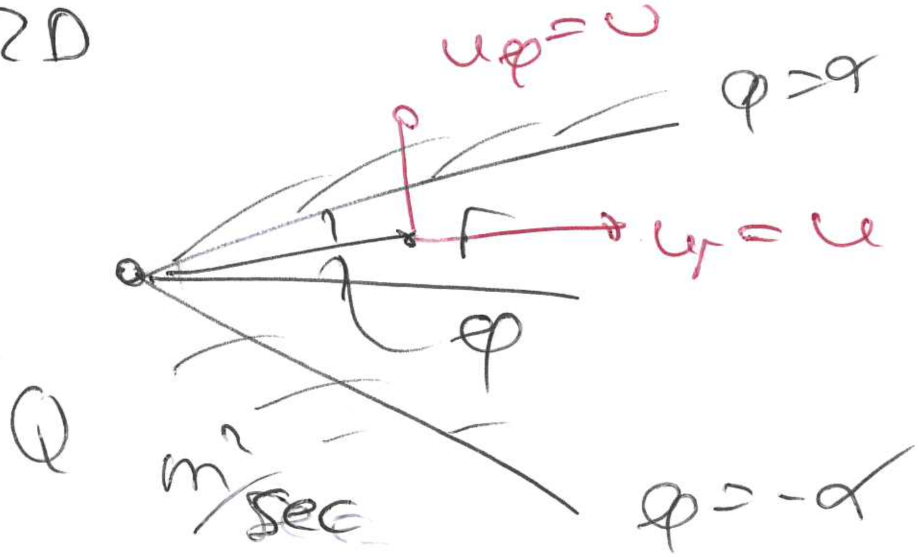
2D



$$\underline{u} = \underline{0}$$

$$\underline{u} = \underline{u}(r, \varphi; \rho, \nu, Q, \alpha)$$

$$\Rightarrow \underline{u} = \frac{\nu}{r}\,\underline{f}\left(\varphi; \alpha, \frac{Q}{\nu}\right)$$

$$= \frac{\nu}{r}\left[f\left(\varphi; \alpha, \frac{Q}{\nu}\right)\underline{e_r} + g\left(\varphi; \alpha, \frac{Q}{\nu}\right)\underline{e_\varphi}\right]$$

Cont.: $$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{1}{r}\frac{\partial v}{\partial \varphi} = 0$$

$$\cancel{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\nu}{r}f(\varphi)\right)} + \frac{1}{r}\frac{\partial}{\partial \varphi}\left[\frac{\nu}{r}g(\varphi)\right] = 0$$

$$\frac{\partial g}{\partial \varphi} = \boxed{\frac{dg}{d\varphi} = 0}$$

BC: $v = 0$ at $\varphi = \pm\alpha$

~~g=0~~ $g=0$ of $\varphi = \pm\alpha$

$\Rightarrow g \equiv 0$

$\Rightarrow$ veloc. is purely radial.

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r - momentum:

$$u\frac{\partial u}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left(\frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \varphi^2} - \frac{u}{r^2}\right)$$

$$u = \frac{\nu}{r}f(\varphi) = \nu r^{-1} f$$

$$\frac{\partial u}{\partial r} = -\nu r^{-2} f$$

$$\frac{\partial^2 u}{\partial r^2} = 2\nu r^{-3} f$$

$$\frac{\partial^2 u}{\partial \varphi^2} = \nu r^{-1} f''$$

$$\underbrace{\left(\frac{\nu}{r}f\right)}_{u}\underbrace{\left(-\nu r^{-2} f\right)}_{\frac{\partial u}{\partial r}} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left(\cancel{-\nu\frac{f}{r^3}} + \cancel{2\nu r^{-3} f} + \frac{\nu}{r^3}f'' \cancel{- \frac{\nu}{r^3}f}\right)$$

$$\boxed{-\frac{\nu^2}{r^3}f^2 = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\nu^2}{r^3}f''} \quad (1)$$

φ-momentum:

$$0 = -\frac{1}{\rho r}\frac{\partial p}{\partial \varphi} + \nu \frac{2}{r^2}\frac{\partial u}{\partial \varphi}$$

$$\frac{\partial p}{\partial \varphi} = \rho\nu \frac{2}{r}\frac{\partial u}{\partial \varphi}$$

$$= \rho\nu \frac{2}{r} \nu \frac{1}{r} f'$$

$$\frac{\partial p}{\partial \varphi} = \frac{2\nu^2\rho}{r^2} f'(\varphi)$$

$$\boxed{p = \rho\frac{2\nu^2}{r^2} f(\varphi) + g(r)} \quad (2)$$

not the same as above!

(2) into (1)

$$-\frac{\nu^2}{r^3} f^2 = -\frac{1}{\rho}\frac{\partial}{\partial r}\Big(\underbrace{\rho 2\nu^2 r^{-2} f(\varphi) + g(r)}_{r}\Big) + \frac{\nu^2}{r^3} f^4$$

Note all terms vary like r^{-3}

$\Rightarrow$ $\frac{dg}{dr}$ must also go like r^{-3}

$$\frac{dg}{dr} = A r^{-3}$$

$$g(r) = -\frac{1}{2} A r^{-2} + C$$

$$g(r) = \frac{B}{r^2} + C$$

$g(r)$ features in the pressure & we can discard arbitrary constants so set $C = 0$.

intb (2):

$$p = \rho\left(\frac{2\nu^2}{r^2}f + \frac{K}{r^2}\right) \quad \text{where } \rho K = B$$

$$p = \rho\left(\frac{2\nu^2}{r^2}f + \frac{B}{\rho r^2}\right)$$

intb (1):

$$-\frac{\nu^2}{r^3}f^2 = -\frac{\partial}{\partial r}\left(\frac{2\nu^2 f + K}{r^2}\right) + \frac{\nu^2}{r^3}f''$$

$$-\frac{\nu^2}{\cancel{r^3}}f^2 = \frac{2(2\nu^2 f + K)}{\cancel{r^3}} + \frac{\nu^2}{\cancel{r^3}}f''$$

ODE for $f(\theta)$ contains arb. constant

$$-\nu^2 f^2 = 2(2\nu^2 f + K) + \nu^2 f''$$

diff. w.r.t θ:

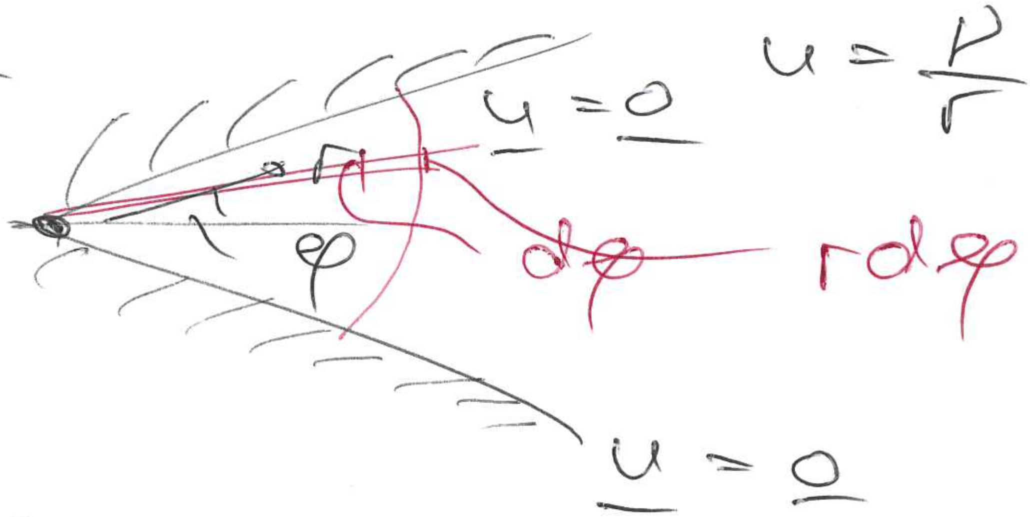
$$-\cancel{\nu^2}2ff' = 4\cancel{\nu^2}f' + \cancel{\nu^2}f'''$$

$$\boxed{f''' + 4f' + 2ff' = 0}$$

3rd order nonlinear ODE.

3BC:

$u = \frac{\nu}{r} f(\varphi)$



$\underline{u} = \underline{0}$ at $\varphi = \pm\alpha$

$$\Rightarrow \boxed{f(\pm\alpha) = 0}$$

Impose volume flux.

$$\boxed{\int_{-\alpha}^{\alpha} u \, r \, d\varphi = -Q}$$