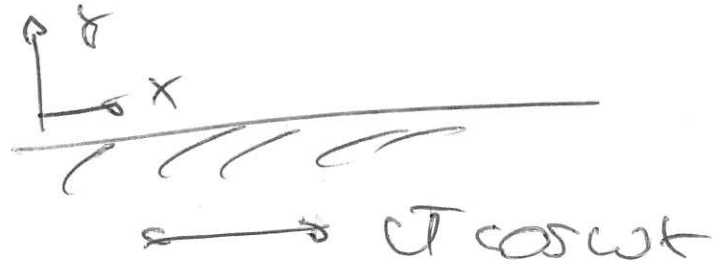
$u = u(y,t)$



$$\boxed{\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2}}$$

BC: $u(y=0,t) = U\cos(\omega t)$

$u \to 0$ as $y \to \infty$

Look for time periodic soln:

$u(y,t) = f(y)\cos(\omega t + \phi(y))$

Better: f complex

$u(y,t) = f(y)\, e^{i\omega t}$ & take real part

into $\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2}$

$i\omega f = \nu f''$ (factor $e^{i\omega t}$ cancelled)

$f'' - \frac{i\omega}{\nu} f = 0$ ODE for $f(y)$

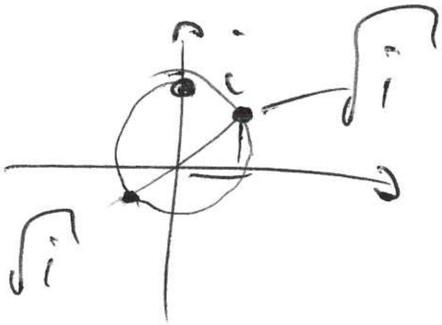
$f \sim e^{\lambda y}$

char poly:

$$\lambda^2 - \frac{i\omega}{\nu} = 0$$

$$\lambda = \pm\sqrt{\frac{i\omega}{\nu}}$$

$\sqrt{i}$?



$$\sqrt{i} = \frac{1}{2}\sqrt{2}\,(1+i)$$

$$\lambda = \pm(1+i)\sqrt{\frac{\omega}{2\nu}}$$

$$f(y) = A\, e^{(1+i)\sqrt{\frac{\omega}{2\nu}}\,y} + B\, e^{-(1+i)\sqrt{\frac{\omega}{2\nu}}\,y}$$

const.

<u>BC:</u> $f(y=0) = U = A + B$

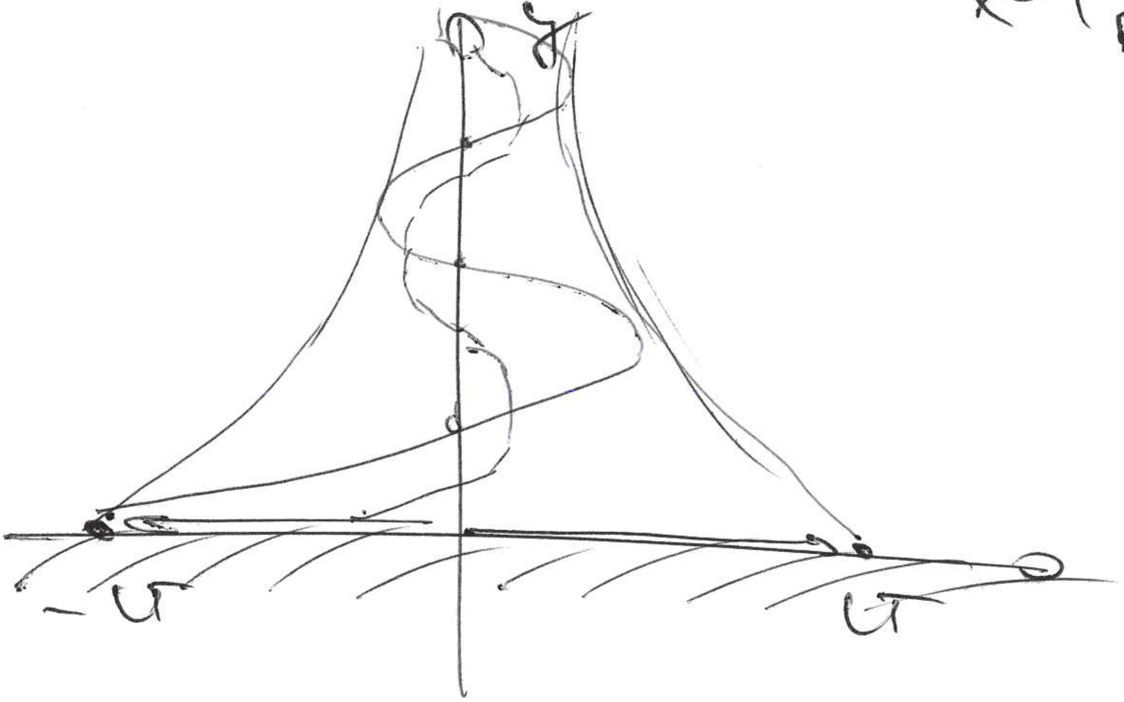
$f \to 0$ as $y \to \infty$: $A = 0$

$\Rightarrow B = U.$

$$u(y,t) = \underbrace{U e^{-(1+i)\sqrt{\frac{\omega}{2\nu}}\,y}}_{f(y)} e^{i\omega t}$$

convert to real:

$$u(y,t) = U e^{-\sqrt{\frac{\omega}{2\nu}}\, y} \cos\Big(\underbrace{\omega t - \sqrt{\tfrac{\omega}{2\nu}}\, y}_{\cancel{\theta}(y)}\Big)$$



$U\cos(\omega t)$

§... Curvilinear coords

So far: everything in Cartesian coords: x_i, u_i

$$\underline{u} = u_1 \underline{e_1} + u_2 \underline{e_2} + u_3 \underline{e_3}$$

Eqns can be transformed to a diff. coord. system

e.g.: $$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x_1^2} + \frac{\partial^2 \phi}{\partial x_2^2}$$

$$x_1 = r\cos\varphi, \quad x_2 = r\sin\varphi \iff r = \sqrt{x_1^2 + x_2^2}, \quad \tan\varphi = \frac{x_2}{x_1}$$

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \varphi^2}$$

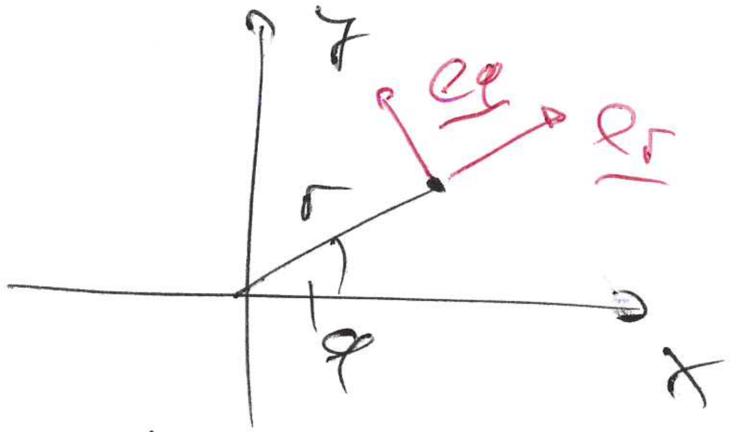
This is for a scalar ϕ only!

N.St. eqns contain vectors!

$$\underline{u} = u_1 \underline{e_1} + u_2 \underline{e_2} + u_3 \underline{e_3}$$

$$= u_r \underline{e_r} + u_\varphi \underline{e_\varphi} + u_z \underline{e_z}$$

but here the basis vectors depend on coords too!



If we differentiate $\underline{u}$ we also have to diff. the basis vectors.

$\Longrightarrow$ A MESS!

(See handout)

Same eqns remain:

$$t_i = \tau_{ij} n_j$$

$$\tau_{ij} = -p\delta_{ij} + 2\mu e_{ij}$$

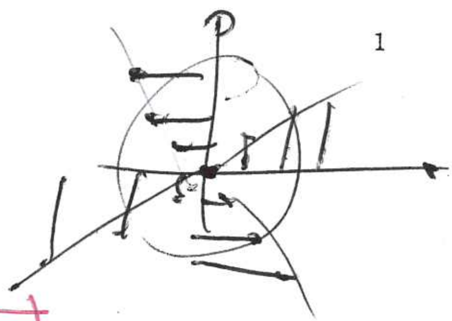
$i, j =$ "r", "φ", "z"

C6

$$u = u_r = 0$$

$$\boxed{v = u_\varphi = \Omega r}$$

$$w = u_z = 0$$



$$\cancel{\frac{\partial u}{\partial t}} + \cancel{u\frac{\partial u}{\partial r}} + \cancel{\frac{v}{r}\frac{\partial u}{\partial \varphi}} + \cancel{w\frac{\partial u}{\partial z}} \boxed{- \frac{v^2}{r} = -\frac{1}{\rho}\frac{\partial P}{\partial r}} + \nu\left[\cancel{\nabla^2 u} - \cancel{\frac{u}{r^2}} - \cancel{\frac{2}{r^2}\frac{\partial v}{\partial \varphi}}\right],$$

$$\cancel{\frac{\partial v}{\partial t}} + \cancel{u\frac{\partial v}{\partial r}} + \cancel{\frac{v}{r}\frac{\partial v}{\partial \varphi}} + \cancel{w\frac{\partial v}{\partial z}} + \cancel{\frac{uv}{r}} = -\frac{1}{\rho r}\frac{\partial P}{\partial \varphi} + \cancel{\nu\left[\nabla^2 v - \frac{v}{r^2} + \frac{2}{r^2}\frac{\partial u}{\partial \varphi}\right]},$$

$$\cancel{\frac{\partial w}{\partial t}} + \cancel{u\frac{\partial w}{\partial r}} + \cancel{\frac{v}{r}\frac{\partial w}{\partial \varphi}} + \cancel{w\frac{\partial w}{\partial z}} = \cancel{-\frac{1}{\rho}\frac{\partial P}{\partial z}} + \cancel{\nu\nabla^2 w},$$

$$\operatorname{div}\mathbf{u} = \cancel{\frac{1}{r}\frac{\partial}{\partial r}(ru)} + \cancel{\frac{1}{r}\frac{\partial v}{\partial \varphi}} + \cancel{\frac{\partial w}{\partial z}} = 0.$$

where

$$\nabla^2 \equiv \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2}.$$