MATH35001: EXAMPLE SHEET[1] III

1.) A 2D flow field is given by $u_1 = 1+x_2^2 \cos x_1$ and $u_2 = 3+Ax_2^3 \sin x_1$. For which value of the constant A is this flow field consistent with the assumption of incompressibility?

2.) The observation of a 2D flow field shows that the velocities along the four edges of the unit square ($x_i \in [0,1]$) are given by

$$\begin{aligned}
\mathbf{u} &= (3+x_2, 4) \quad \text{on} \quad x_1 = 0, \\
\mathbf{u} &= (3+x_2+x_2^3, 7-\frac{1}{2}x_2^4) \quad \text{on} \quad x_1 = 1, \\
\mathbf{u} &= (3, 4+3x_1^2) \quad \text{on} \quad x_2 = 0, \\
\mathbf{u} &= (4+x_1^2, 4-\frac{1}{2}x_1+3x_1^2) \quad \text{on} \quad x_2 = 1.
\end{aligned}$$

Is the fluid incompressible? (Think carefully about this!).

3.) Consider the infinitesimal tetrahedron used in the derivation of the stress tensor and show that

$$\mathbf{n}_i ds_i + \mathbf{n} ds = 0$$

where the $\mathbf{n}_i$ are the outside unit normal vectors on the three faces on which $x_i = const.$, the ds_i are their areas, $\mathbf{n}$ is the outside unit normal vector on the fourth (general) face and ds is its area. [Hint: Express the areas via cross products of the three vectors forming the tetrahedron's edges on the coordinate axes].

Coursework

Please exchange your solution to questions 1 and 2 with your "marking buddy" and assess each other's work, using the master solution made available on the course webpage (probably in week 4).

[1] Any feedback to: *M.Heil@maths.man.ac.uk*