MATH35001: EXAMPLE SHEET¹ VI

- 1.) An infinite pipe of circular cross-section and radius b has a rigid circular rod of radius a lying along its axis which coincides with the z-axis of a circular cylindrical coordinate system. The annular region between the rod and the walls of the pipe is filled with a viscous fluid of dynamic viscosity μ . The rod is drawn along the length of the pipe with constant speed $U \mathbf{e}_z$ and the fluid is subject to an axial pressure gradient $\nabla p = G \mathbf{e}_z$.
 - (i) Find the flow field in the pipe and the drag (per unit axial length) exerted by the fluid onto the rod.
 - (ii) Determine the value of $G = G_0$ for which the drag is equal to zero. Sketch velocity distributions for G = 0, G < 0 and $G = G_0$.
- 2.) Fluid is contained in an infinitely long pipe of square cross section whose edges of length a are situated at $x = \pm a/2$ and $y = \pm a/2$. The fluid is driven through the pipe by an applied pressure gradient $\nabla p = G \mathbf{e}_z$. Determine the flow field in the pipe. You might want to follow these steps:
 - (i) Choose a particular solution which is independent of x and fulfills the boundary conditions at $y = \pm a/2$.
 - (ii) Use separation of variables for the homogeneous solution; choose the sign of the constant such that the y-dependence involves trigonometric functions.
 - (iii) Determine the unknown coefficients from the boundary conditions at $x = \pm a/2$. Hint: Expand the particular solution into a Fourier series. You can use the following results:

$$\int_{-a/2}^{a/2} \cos\left(\frac{(2\,m-1)\,\pi\,y}{a}\right) \,\cos\left(\frac{(2\,n-1)\,\pi\,y}{a}\right) \,dy = \begin{cases} a/2 & \text{for } n = m\\ 0 & \text{for } n \neq m \end{cases}$$
$$\int_{-a/2}^{a/2} \frac{G}{2\mu} (y^2 - \frac{1}{4}\,a^2) \cos\left(\frac{(2\,m-1)\,\pi\,y}{a}\right) \,dy = 2\frac{a^3\,G\,(-1)^m}{(2\,m-1)^3\,\pi^3\,\mu}$$

Course work

Please exchange your solution to question 1 with your "marking buddy" and assess each other's work, using the master solution made available on the course webpage (probably in week 8).

¹Any feedback to: M.Heil@maths.man.ac.uk