MATH35001: EXAMPLE SHEET[1] VI

1.) An infinite pipe of circular cross-section and radius b has a rigid circular rod of radius a lying along its axis which coincides with the z-axis of a circular cylindrical coordinate system. The annular region between the rod and the walls of the pipe is filled with a viscous fluid of dynamic viscosity μ. The rod is drawn along the length of the pipe with constant speed $U\ \mathbf{e}_z$ and the fluid is subject to an axial pressure gradient $\nabla p = G\ \mathbf{e}_z$.

(i) Find the flow field in the pipe and the drag (per unit axial length) exerted by the fluid onto the rod.

(ii) Determine the value of $G = G_0$ for which the drag is equal to zero. Sketch velocity distributions for $G = 0, G < 0$ and $G = G_0$.

2.) Fluid is contained in an infinitely long pipe of square cross section whose edges of length a are situated at $x = \pm a/2$ and $y = \pm a/2$. The fluid is driven through the pipe by an applied pressure gradient $\nabla p = G\ \mathbf{e}_z$. Determine the flow field in the pipe. You might want to follow these steps:

(i) Choose a particular solution which is independent of x and fulfills the boundary conditions at $y = \pm a/2$.

(ii) Use separation of variables for the homogeneous solution; choose the sign of the constant such that the y-dependence involves trigonometric functions.

(iii) Determine the unknown coefficients from the boundary conditions at $x = \pm a/2$. Hint: Expand the particular solution into a Fourier series. You can use the following results:

$$\int_{-a/2}^{a/2} \cos\left(\frac{(2\,m-1)\,\pi\,y}{a}\right)\cos\left(\frac{(2\,n-1)\,\pi\,y}{a}\right)\,dy = \begin{cases} a/2 & \text{for } n = m \\ 0 & \text{for } n \neq m \end{cases}$$

$$\int_{-a/2}^{a/2} \frac{G}{2\mu}\left(y^2 - \frac{1}{4}\,a^2\right)\cos\left(\frac{(2\,m-1)\,\pi\,y}{a}\right)\,dy = 2\,\frac{a^3\,G\,(-1)^m}{(2\,m-1)^3\,\pi^3\,\mu}$$

Coursework

Please exchange your solution to question 1 with your "marking buddy" and assess each other's work, using the master solution made available on the course webpage (probably in week 8).

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