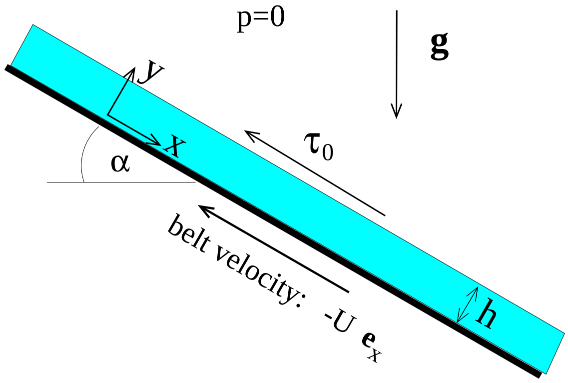
MATH35001: EXAMPLE SHEET[1] V

1.) The figure below shows a film of Newtonian incompressible fluid on an inclined belt which is moving with constant velocity U. The no-slip condition applies on the surface of the belt, i.e. the fluid particles on the belt move with velocity $\mathbf{u}(y=0) = -U\mathbf{e}_x$. Gravity acts vertically downwards and a strong wind exerts a tangential shear stress τ_0 in the negative x-direction onto the surface of the fluid. You can assume that the flow is steady and unidirectional and that the film thickness h is constant along the belt (and given). The air pressure at the free surface is constant and given by $p = 0$.

(i) Determine the velocity field and the pressure distribution in the fluid.

(ii) Determine the volume flux Q (per unit width of the belt) in the positive x-direction, i.e. evaluate $Q = \int_0^h u(y)\ dy$.

(iii) Now consider the case $U = 0$, i.e. a stationary belt. Determine the critical value τ_0^{crit} of the shear stress τ_0 for which the volume flux becomes negative (in other words, for $\tau_0 > \tau_0^{crit}$ *overall* the fluid flows 'up the hill'). Sketch the velocity distributions for the cases $\tau_0 = 0$ and $\tau_0 = \tau_0^{crit}$ (still assuming that $U = 0$).



2.) Fluid is confined between two infinite parallel plates at $y = 0$ and $y = h$. An externally applied pressure gradient $\nabla p = G\mathbf{e}_x$ drives the fluid in the x-direction. The plates are porous and fluid is driven through the top surface at $y = h$ with a uniform normal velocity V and leaves the bottom wall at the same uniform velocity such that $\mathbf{u} = -V\mathbf{e}_y$ at $y = 0$ and $y = h$. You can assume that there is no motion in the z-direction (i.e. $w = 0$) and that all quantities are independent of z.

(i) Explain why $\mathbf{u}(x, y, t) = (u(y), -V)$ is a plausible guess for the velocity field.

(ii) Show that the velocity field assumed in (i) is consistent with the 2D Navier Stokes equations and the equation of continuity and that the only non-trivial equation is given by

$$\nu \frac{\partial^2 u}{\partial y^2} + V \frac{\partial u}{\partial y} = \frac{G}{\rho}.$$

(iii) Solve this equation subject to the no-slip condition for u on the top and bottom walls [Hint: The constant term on the RHS is a singular form since a constant function is already contained in the solution of the homogeneous equation. Therefore, the particular solution must have the form 'constant $\times y$'].

Coursework

Please exchange your solution to question 1 with your "marking buddy" and assess each other's work, using the master solution made available on the course webpage (probably in week 7).

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