MATH35001: EXAMPLE SHEET[1] III

1.) A 2D flow field is given by $u_1 = 1 + x_2^2 \cos x_1$ and $u_2 = 3 + A x_2^3 \sin x_1$. For which value of the constant A is this flow field consistent with the assumption of incompressibility?

2.) The observation of a 2D flow field shows that the velocities along the four edges of the unit square ($x_i \in [0, 1]$) are given by

$$\begin{aligned}
\mathbf{u} &= (3 + x_2, 4) \quad \text{on} \quad x_1 = 0, \\
\mathbf{u} &= (3 + x_2 + x_2^3, 7 - \frac{1}{2} x_2^4) \quad \text{on} \quad x_1 = 1, \\
\mathbf{u} &= (3, 4 + 3x_1^2) \quad \text{on} \quad x_2 = 0, \\
\mathbf{u} &= (4 + x_1^2, 4 - \frac{1}{2} x_1 + 3x_1^2) \quad \text{on} \quad x_2 = 1.
\end{aligned}$$

Is the fluid incompressible? (Think carefully about this!).

3.) Consider the infinitesimal tetrahedron used in the derivation of the stress tensor and show that

$$\mathbf{n}_i ds_i + \mathbf{n} ds = 0$$

where the $\mathbf{n}_i$ are the outside unit normal vectors on the three faces on which $x_i =$ *const.*, the ds_i are their areas, $\mathbf{n}$ is the outside unit normal vector on the fourth (general) face and ds is its area. [Hint: Express the areas via cross products of the three vectors forming the tetrahedron's edges on the coordinate axes].

Coursework

Please hand in the solution to questions 1 and 2 by 12 noon on Thursday. Please place them into the envelope at the door of Prof. Heil's office (room 2.224 in the Alan Turing building).

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