Chapter 7

Dimensional Analysis and Scaling

7.1 Dimensional Analysis and Scaling

- See the copies of the OHP transparencies.

7.2 Similarity Solutions

- Many phenomena in fluid mechanics exhibit self-similar behaviour. A simple example for this is given by a velocity profile $u(y,t)$ which has the same 'shape' for all values of time t. In that case, the solution must have the special form

$$u(y,t) = a(t)\ f\left(\frac{y}{b(t)}\right), \tag{7.1}$$

  where the function $a(t)$ changes the 'amplitude' of $u(y,t)$ while $b(t)$ provides a time-dependent scaling for the y-coordinate, making the velocity profile 'wider' (for $b(t) > 1$) or 'narrower' (for $b(t) < 1$).

- The existence of similarity variables is also familiar from traveling wave problems in which the solution has the form $u(y,t) = f(y - Ut)$. This solution represents a wave of shape $f(y)$ traveling in the positive y-direction with velocity U. The traveling wave coordinate $\eta = y - Ut$ plays the role of a similarity variable.

- In general, similarity solutions are characterised by the requirement that at least one independent variable only occurs in a certain combination with other independent variables. This often simplifies the mathematical analysis, for instance, by transforming PDEs into ODEs.

- The search for suitable similarity variables is often aided by dimensionality considerations.

- The choice of the similarity variable is usually not unique. For instance, a function $f(\eta)$ could also be regarded as a function $F(\eta^2)$ – obviously, both η and η^2 are perfectly acceptable choices. Typically, one tries to keep the similarity variable linear in the spatial coordinate, as in (7.1) where $\eta = y/b(t)$.

- Similarity solutions only 'work' if the boundary and initial conditions can also be formulated in terms of the similarity variable. If this is not the case, the similarity 'solution' might (!) still represent a useful approximation to the exact solution.