Chapter 5

Parallel Flows

5.1 The parallel flow equations

- The main difficulty in the solution of the Navier Stokes equations arises from their nonlinearity. There are, however, situations in which the nonlinear terms vanish identically.
- This happens (for instance) if the flow is *unidirectional*. If this is the case then we can choose our coordinate system such that the x-axis is aligned with the flow and the velocity field has the form $\mathbf{u} = u(x, y, z, t)\ \mathbf{e}_x$.
- Inserting this assumption into the Navier Stokes and continuity equation shows that this is only possible if
$$\mathbf{u} = u(y, z, t)\ \mathbf{e}_x, \tag{5.1}$$
i.e. if the velocity is independent of the streamwise coordinate.
- The flow governed by the following three linear equations
$$\rho \frac{\partial u}{\partial t} = \rho F_x - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right), \tag{5.2}$$
$$0 = \rho F_y - \frac{\partial p}{\partial y} \tag{5.3}$$
and
$$0 = \rho F_z - \frac{\partial p}{\partial z}. \tag{5.4}$$

5.2 The parallel flow equations without body force

- If the body force vanishes (i.e. $F_x = F_y = F_z = 0$) it can be shown that $p = p(x, t)$ and the pressure gradient has to have the form
$$\nabla p = G\ \mathbf{e}_x \tag{5.5}$$
where G is a constant. (If the pressure gradient has any other form, then no parallel flow is possible).
- In this case, the only non-trivial equation is the x-momentum equation which becomes
$$\rho \frac{\partial u}{\partial t} = -G + \mu \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right). \tag{5.6}$$