MT4261: EXAMPLE SHEET[1] II (for Sept. 30, 1999)

1.) A 2D flow field is given by $u_1 = ax_2$ and $u_2 = ax_1$.

a) Sketch the flow field (begin by considering the velocity vectors on the coordinate axes and on the main diagonals).

b) Determine the trajectories $x_i^p(t)$ of particles which are at position $x_i = X_i$ at time $t = 0$ by integrating the equations of motion $\partial x_i^p(t)/\partial t = u_i(x_j^p(t), t)$.

c) Determine the acceleration of material particles directly from their trajectories (i.e. by evaluating $a_i = \partial^2 x_i^p(t)/\partial t^2$).

d) Compare this to the result obtained by using the material derivative $a_i = Du_i/Dt$.

e) What is the divergence of the flow field?

2.) A 2D flow field is given by $u_1 = Ux_2$ and $u_2 = 0$.

a) Sketch the flow field in the region $x_2 \in [0, 1]$.

b) Determine the rate of strain and the rate of rotation tensors ϵ_{ij} and ω_{ij}.

c) Sketch the deformation of a small rectangular fluid element whose edges of lengths δx_1 and δx_2 are parallel to the x_1 and x_2 axes, respectively. (Hint: Sketch the corners of the fluid element at time t and at time $t + \delta t$). Relate this to the interpretation of the rate of strain and rate of rotation tensors.

Coursework

Please hand in the solution to question 2 by 12 noon on Thursday, Oct 7, 1999. Please place them into the file in Dr. Heil's pigeonhole in the general office on the 4th floor.

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