MT4261: EXAMPLE SHEET[1] I (for Sept. 23, 1999)

1.) Which of these equations in index notation are valid? Remember the summation convention!

- **a)** $c = a_i b_i$
- **b)** $c = a_{ij} b_i$
- **c)** $c_i = a_{ij} b_i$
- **d)** $c_i = a_{ij} b_j$
- **e)** $c_i = a_{ji} b_j$
- **f)** $\sigma_{ij} = \alpha_{ij} T + E_{ijkl} e_{kl}$
- **g)** $\sigma_{ij} = \alpha_{kl} T_i + E_{ijkl} e_{ij}$
- **h)** $k_{ijkl} = a_i b_{kl} c_{njm} d_{mn} + e_{ik} e_{jn} f_{nl}$

2.) Using a comma to denote partial differentiation (e.g. $\partial u/\partial x_2 = u_{,2}$), transform the following expressions into index notation:

- **a)** $\nabla u(x_1, x_2, x_3)$
- **b)** $\underline{\mathbf{A}} = \nabla \mathbf{u}(x_1, x_2, x_3)$ (Hint: $\underline{\mathbf{A}}$ is a matrix).
- **c)** $\nabla \cdot \mathbf{u}(x_1, x_2, x_3) = f(x_1, x_2, x_3)$
- **d)** $\nabla^2 u(x_1, x_2, x_3) = f(x_1, x_2, x_3)$
- **e)** $\nabla^2 \mathbf{u}(x_1, x_2, x_3) = \mathbf{f}(x_1, x_2, x_3)$

3.) **a)** Show that $\frac{\partial x_i}{\partial x_j} = \delta_{ij}$.

- **b)** Show that $\delta_{ii} = 3$.
- **c)** For arbitary vectors u_i and v_i show that

$$S_{ij} = u_i v_j + u_j v_i$$

is symmetric (i.e. $S_{ij} = S_{ji}$) and that

$$T_{ij} = u_i v_j - u_j v_i$$

is antisymmetric (i.e. $T_{ij} = -T_{ji}$).

> *Coursework*
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> Please hand in the solution to question 2 by 12 noon on Thursday, Sept 30, 1999. Please place them into the file in Dr. Heil's pigeonhole in the general office on the 4th floor.

[1] Any feedback to: *M.Heil@maths.man.ac.uk*