1.) Two infinitely long solid plane walls are smoothly hinged at their intersection. The wedge shaped region between the plates is filled with viscous fluid. One wall is fixed while the other wall rotates slowly with angular velocity Ω (see Fig. 1). The current opening angle between the plates is given by α. You can assume that the flow is two-dimensional and governed by Stokes' equations.

(i) State the governing PDE and express the velocity boundary conditions in terms of the streamfunction $\psi(r,\varphi)$. [Hints: (a) integrate w.r.t. to r where possible; (b) use the fact that ψ is only determined up to an arbitrary constant to obtain as many homogeneous BCs as possible].

(ii) Use linearity and dimensionality arguments to show that the streamfunction $\psi(r,\varphi)$ must have the form

$$\psi(r,\varphi) = \Omega r^2 f(\varphi).$$

Don't just check the dimensions of this expression; show that *no other form is possible.*

(iii) Show that $f(\varphi)$ satisfies the ODE

$$\frac{d^2}{d\varphi^2}\left(4 + \frac{d^2}{d\varphi^2}\right) f(\varphi) = 0.$$

(iv) Solve the PDE subject to the BCs derived in (i) to show that

$$f(\varphi) = \frac{1}{4(\tan\alpha - \alpha)}[\tan\alpha\ (1 - \cos 2\varphi) + \sin 2\varphi - 2\varphi]$$

[Hint: The identity $(1 - \cos 2\alpha) = 2\ \sin^2\alpha$ might be useful].

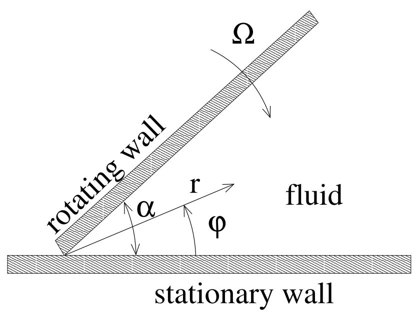

Figure 1: Sketch of a 2D squeezing flow in a wedge.

Coursework
Please hand in the solutions to question 1 by 12 noon on Friday. Please place them into the envelope at the door of Dr. Heil's office (room 1.05 in the Lamb building).

This is the last bit of coursework. Good luck in the exam!

[1] Any feedback to: *M.Heil@maths.man.ac.uk*