1.) Consider a 2D channel of length L whose height $h(x,t)$ varies slowly in the x-direction such that $\partial h/\partial x = O(h_0/L) \ll 1$, where h_0 is the average width of the channel. The upper wall is moving with velocity $U\mathbf{e}_x$. The scaling arguments used in the lecture showed that to leading order (i.e. within an error of order $O((h_0/L)^2)$ or $O(Re\ h_0/L)$ – whichever is larger), the x-component of the momentum equation can be approximated by

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2}.$$

Show that to the same order of approximation, the y-component of the momentum equation can be simplified to

$$\frac{\partial p}{\partial y} = 0.$$

2.) Fig. 1 shows a sketch of a slider bearing. The upper wall is stationary while the bottom wall moves with velocity $U\mathbf{e}_x$. A thin film of viscous fluid separates the two solid surfaces. You can assume that the Reynolds number and the wall slopes are small enough to justify the lubrication theory approximation.

(i) Use the continuity of flux to determine an expression for the pressure distribution in the lubrication film, assuming that the fluid pressure at both ends of the bearing is equal to the atmospheric pressure, i.e. $p(0) = p(L) = p_0$.

(ii) Evaluate the pressure distribution (which contains integrals involving the gap width $h(x)$) for the special case of a linearly varying lubrication gap for which $h(x) = h_L + (h_R - h_L)\ x/L$, where $|(h_R - h_L)/L| \ll 1$. Hints:

(i) $$\int_0^x h(s)^{-N} ds = \frac{L}{(1-N)(h_R - h_L)} \left(h(x)^{1-N} - h_L^{1-N} \right)$$

(ii) The final result is: $$\frac{p(x) - p_0}{6\mu U L} = \frac{(h_R - h(x))(h_L - h(x))}{(h_R^2 - h_L^2)\ h(x)^2}$$

(iii) Consider the cases of a widening ($h_R > h_L$) and narrowing ($h_R < h_L$) lubrication gap. In which case can the upper block support a downward force F (as shown in Fig. 1)?

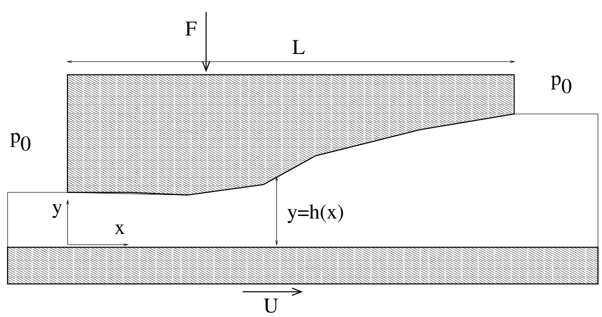

Figure 1: Sketch of a slider bearing.

Coursework

Please hand in the solutions to question 2 by 12 noon on Friday. Please place them into the envelope at the door of Dr. Heil's office (room 1.05 in the Lamb building).

[1]Any feedback to: *M.Heil@maths.man.ac.uk*